# Inflation, the Skill Premium and the labor share: An empirical and theoretical analysis

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# Inflation, the Skill Premium and the labor share: An empirical and theoretical analysis*

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**Abstract**

We develop an overlapping generations endogenous growth model with cash-in-advance constraints for (i) consumers and (ii) R&D firms which is consistent with an effect of inflation on the skill-premium labor share. Inflation decreases the skill premium in both cases and decreases the labor share through (i) which it increases through (ii). The newly described effect of inflation on the labor share is consistent with empirical evidence for a short-run effect.

**Keywords:** inflation, labor share, human capital

**JEL codes:** E24, J64, L11, O33

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# 1 Introduction

Despite the decline in labor share over recent decades and the increased instability of inflation in many countries worldwide, the relationship between these two factors has been largely overlooked. Empirical evidence so far contradicts predictions from new Keynesian models. For example, Cantore et al. (2020) present evidence that monetary policy tightening led to an increase in labor share and a decrease in real wages during the Great Moderation period in the United States, the Euro Area, the United Kingdom, Australia, and Canada—opposite to what theoretical models predict. In contrast, Kaplan and Zoch (2020) and Chu (2020) show that when monetary policy shocks and markup shocks are positively (or negatively) correlated, monetary contraction increases (or decreases) both total labor share and price markup. As a result, the labor share is countercyclical in the first case and procyclical in the second.

Using Calvo-style models of nominal rigidities, Gaí et al. (2001) demonstrate that their model explains the simultaneous decline in inflation and labor share in the Euro Area. However, Lawless and Whelan (2011) show that, under realistic parameter values, the model fails to explain the joint behavior of inflation and labor share in Europe. Furthermore, they find that the model performs poorly when applied to sectoral data, consistently producing negative estimated coefficients on the labor share across various inflation specifications.

Regardless of the causal link studied, and despite the lack of a clear empirical conclusion, previous research has relied on short-run models. In this study, we aim to examine the relationship between inflation and labor share in both the short and long run. To achieve this, we employ an endogenous growth model to characterize both steady-state behavior and transitional dynamics.

Recent literature, beginning with Funk and Kromer (2010), Chu and Lai (2013), and Chu and Cozzi (2014), has explored the long-run effects of inflation within endogenous growth models, highlighting the non-neutrality of monetary policy, despite its quantitatively small effects. Their approach uses cash-in-advance (CIA) constraints on consumption expenditures and capital costs to analyze the effects of inflation on growth (e.g., Chu et al., 2019; Gil and Iglesias, 2020) and inequality (e.g., Afonso and Lima, 2023; Afonso and Sequeira, 2023). Also regarding the effect of inflation on the skill premium the literature is not consensual. While Afonso and Lima (2023) point out for a rise in the skill premium due to inflation according to some assumption of different CIAs in the model, Afonso and Sequeira (2023) point out for a negative relationship between both variables, Chu et al. (2019) find an inverted U-shaped relationship between inflation and inequality meaning that inflation increases inequality only for lower levels of inflation. Finally, Hu et al. (2024) argues for a U-shaped relationship for sufficiently big countries, indicating that inflation may decrease inequality for lower level of inflation. As Afonso and Sequeira (2023) and Hu et. al. (2024) focus on open economies setup, the (theorectical) sign of the effect seem to depend on the openness of the economy.

Noteworthly, none of these studies examine the impact of inflation on labor share, despite it being a natural next step. Our research aims to fill this gap.

We present new evidence on the relationship between inflation and the labor share. This tend to show a cointegration relationship between both variables and also other controls. However, our results show that when significant effect is detected it is negative and only in the short-run. In the long-run, inflation tend to be non-significantly related to the labor share, whatever the set of controls considered. Then, we investigate the theoretical effects of inflation on skill-premium and the labor share through a (small open economy) overlapping generations endogenous growth model with cash-in-advance (CIA) constraints affecting either consumers or R&D firms. Inflation increases the supply of skilled workers through cash-in-advanced consumers by creating more incentives for low-skilled workers to pursue education and avoid its negative consequences, which are felt more strongly by this type of worker. This, in turn, under certain conditions, leads to a higher share of scientists and a lower skill premium and labor share. Through R&D firms facing liquidity constraints, it decreases the demand for scientists and skilled workers, leading to the same effect in the skill premium and the opposite in the labor share.

The paper is organized as follows. In the next Section we present empirical evidence on panel data relating the labor share and inflation. In Section 3 we present the model and the theoretical results. Section 4 concludes.

# 2 Some Empirical Evidence

## 2.1 Descriptive statistics

Our dataset includes yearly observations for 38 countries between 2000 and 2019: Australia, Austria, Belgium, Canada, Chile, Costa Rica, Czechia, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Israel, Italy, Japan, Latvia, Lithuania, Luxembourg, Mexico, Netherlands, New Zealand, Norway, Poland, Portugal, Korea, Slovakia, Slovenia, Spain, Sweden, Switzerland, United Kingdom, United States.

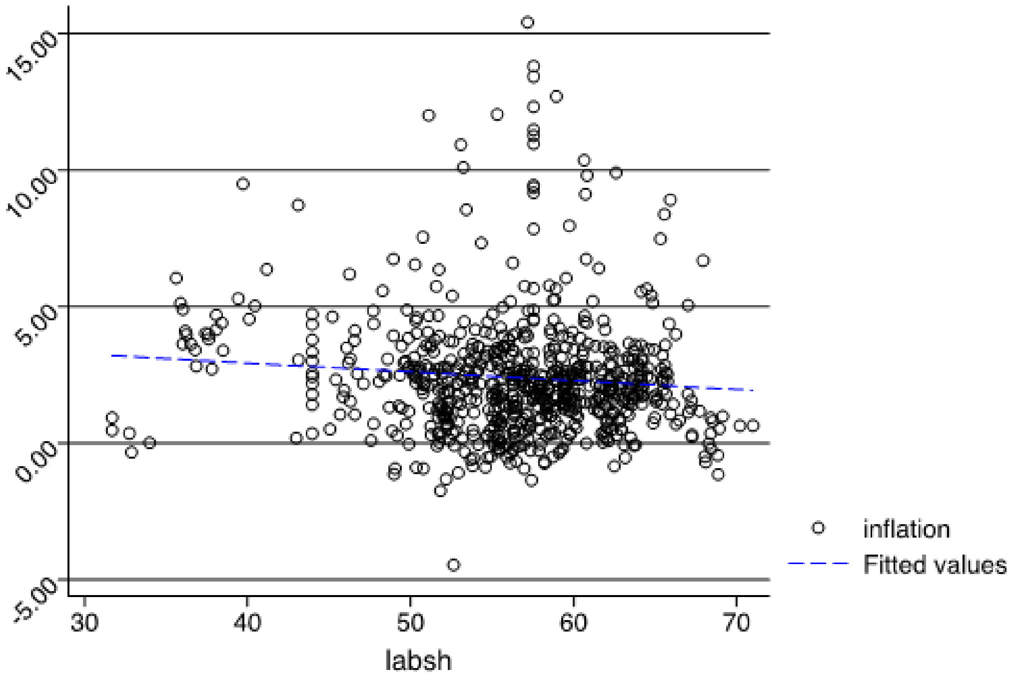
The main variables used are the labor share taken from the Penn World Table (PWT) 10.01 (Feenstra et al., 2015), inflation (based on consumer prices) which source was the World Bank Database, and unemployment and taken from the same source as inflation. We also use investment in physical capital (k) from the PWT and human capital (h) from the World Bank Database. Table 1 shows descriptive statistics.

For the full dataset we can observe the negative correlation between our two variables of interest (Figure 1).

Table 1: Descriptive statistics

| Variables | N | mean | sd | min | max | variance | median | skewness | kurtosis |
|---|---|---|---|---|---|---|---|---|---|
| labor share | 720 | 56.44 | 6.556 | 31.68 | 71.00 | 42.98 | 42.98 | -0.934 | 4.547 |
| inflation | 720 | 2.407 | 2.245 | -4.448 | 15.40 | 5.038 | 5.038 | 1.936 | 9.347 |

Figure 1: Scatterplot of all the datapoints in the database showing a negative correlation between the two main series.




Further tests presented in the Appendix. After showing that most variables are at least I(1), a cointegration test shows there is evidence of the existence of a cointegration.[1] Building on these results we move into the estimation of a model with a panel ARDL specification, following (Pesaran & Smith, 1995; Pesaran et al., 1999), as described in the following section.

## 2.2 Long-Run and Short-Run relationships

In the empirical application we make use of a Mean Pooled Group estimator to analyze the link between inflation and the labor share in the short and long-run. A significantly negative coefficient for the lag of the labor share is a test for cointegration.

The baseline specification we estimate is the following:

$$\Delta \text{labor share}_{i,t} = \beta_0 + \beta_1 \ \Delta \text{inflation}_{i,t} + \beta_2 \Delta u_{i,t} + \gamma \left(\text{labor share}_{i,t-1} - \alpha_1 \text{inflation}_{i,t-1} - \alpha_2 u_{i,t-1}\right) + \epsilon_{i,t} \quad (1)$$

where labor share and inflation are self explanatory and $u$ is a measure of unemployment. In alternative specifications we also use investment in physical capital (k) and human capital (h) and a trend as controls. Unemployment is used as a control due to its short-run relationship with inflation (through the Phillips curve) and the potential effect in equilibrium wages thus potentially affecting the labor share. The

[1]It is important to note that all the considered variables – except human capital (h)–, in first differences are I(0).

inclusion in some specification of investment and human capital are due to their role as engines of growth (the first mainly in the short run) and thus potentially affecting the labor share.

Table 2: Summary of Empirical results

| | (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|---|
| $\gamma$ | -0.596*** | -0.798*** | -1.042*** | -0.158 | -0.581*** | -0.816*** |
| | (0.067) | (0.072) | (0.144) | (1.280) | (0.087) | (0.087) |
| $\Delta inflation_{i,t}$ | -0.050 | -0.103 | -0.380* | -4.856* | -0.088 | -0.173* |
| | (0.082) | (0.081) | (0.227) | (2.557) | (0.091) | (0.1000) |
| $inflation_{i,t-1}$ | 0.037 | -0.053 | -0.167 | 1.092 | 0.150 | -0.018 |
| | (0.101) | (0.116) | (0.318) | (4.932) | (0.132) | (0.155) |
| trend | No | Yes | No | Yes | No | Yes |
| Controls | $u$ | $u$ | $u, hc, \Delta gfcg$ | $u, hc, \Delta gfcf$ | $u, \Delta gfcf$ | $u, \Delta gfcf$ |

Note: unemployment is included as control in Columns (1) and (2); unemployment, human capital and investment (in the short-run) are included as controls in Columns (3) and (4); unemployment and investment (in the short-run) are included as controls in Columns (5) and (6). In columns (2), (4) and (6) a trend is included. All regressions include a constant which is not shown. S.d are included below each coefficient in parenthesis. * indicates significance at $<10\%$ level; ** indicates significance at $<5\%$ level; *** indicates significance at $<1\%$ level.

There are three main takeaways from the results in Table 1. First there is evidence of cointegration since the coefficient of the lag of the dependent variable is negative and statistically significant. Second the long-run effect of inflation represented by the coefficients of inflation$_{i,t-1}$ is always non-significant. Third the short-run effect of inflation represented by the coefficients of $\Delta$inflation$_{i,t}$ are either non-significant or statistically significant with a negative sign. This pattern is repeated also in several experiments we did which are not shown in the table.

In the next Section we develop a OLG model which reassess the effect of inflation in the skill premium and the newly addressed effect of inflation in the labor share.

# 3 An Overlapping generation model with money demand

We now analyze the theoretical impacts of inflation in the skill premium and the labor share. To do so we consider as baseline model a two-generation overlapping generations model (OLG) in a similar manner as other approaches in the literature (e.g., Prettner and Strulik, 2020), where time evolves discretely with each time corresponding to the period of a generation. In subsection 3.1, we develop the baseline model with money demand introduced through the assumption that individuals have cash-in-advance (CIA) constraints in consumption and analyze the impacts of inflation in this context. In subsection 3.2 we introduce money demand by assuming that firms in the R&D sector face CIA constraints, explain the main differences relative to previous extension and analyze the impacts of inflation in this context.

## 3.1 Cash-in-advance constraints in consumption

### 3.1.1 Individuals

We assume that individuals live for two periods, working age and old age, and are each randomly assigned an ability level from a distribution between 0 and 1. Before working age, they become aware of their ability level and choose accordingly between two mutually exclusive alternatives regarding how to spending time in working age: (i) if they have an ability level below some threshold $\overline{a}$, they spend all the available time effectively working, (ii) otherwise, they spend a fraction $\eta$ of the available time studying to increase their skills, by enrolling in tertiary education and obtaining high-skilled human capital, and the remaining time, $1-\eta$, working.

Those who opt for (i) are of type $j = L$ or low-skilled and those that opt for (ii) are of type $j = H$ or high-skilled. Considering that the total number of supplied working hours is $L_t^S$, and that the number of hours supplied by low-skilled and high-skilled workers are given, respectively, by $L_{L,t}^S$ and $L_{H,t}^S$, the previous assertion that the corresponding population shares of supplied hours are as follows:

$$l_{L,t}^S \equiv \frac{L_{L,t}^S}{L_t^S} = \underline{a},\ l_{H,t}^S \equiv \frac{L_{H,t}^S}{L_t^S} = 1 - \underline{a}\,, l_{L,t}^S + l_{H,t}^S = 1. \tag{2}$$

We assume that choice (ii) implies an a disutility from the required effort. Therefore, the individual must compare the advantages of studying, which are higher wages, with the disadvantages, which are enduring a period without receiving any wages and a disutility from the required effort. Considering this, and that individuals experience utility from consumption in working age and old age, the lifetime utility of individuals of type $j$ in period $t$ is given by

$$u_{j,t} = \log\left(c_{j,t}\right) + \beta \log\left(d_{j,t+1}\right) - \mathbb{1}_{[j=H]}\omega\left(a\right), \tag{3}$$

where $c_{j,t}$ and $d_{j,t+1}$ are the consumption levels of individuals of type $j$ at time $t$ (when they are young adults) and time $t+1$ (when they are old), respectively, $\beta$ is the discount factor, $\mathbb{1}_{[j=H]}$ is the indicator function which assumes the value of one for individuals of type $j$ and $\omega\left(a\right)$ captures the disutility from studying. Regarding the latter, we assume that (i) is a negative function of ability, i.e., $\frac{\partial \omega(a)}{\partial a} < 0$, to capture the fact that that the less skill an individual has the higher the difficulty of obtaining a college degree, (ii) individuals with an ability level below some threshold $a_{min}$ experience infinite disutility from studying, i.e., $\omega\left(a\right) = \infty$ for $a < a_{min}$, which captures the notion that not all individuals are effectively able to obtain a college degree due to implying a prohibitive effort. These features are conveniently captured by the following function:

$$\omega\left(a\right) \equiv \begin{cases} \infty & a \leq a_{min} \\ \theta \log\left(\frac{\aleph}{a - a_{min}}\right), & a > a_{min} \end{cases}, \tag{4}$$

where $\theta$ and $\aleph$ are parameters used for calibration of the ability function further ahead. Following Prettner and Strunik (2020), we assume for simplification that $\theta = 1 - \beta$.

In period $t$, the adult $j$ earns expected wages of $w_{j,t}(1 - \eta_j)$ per unit of time, where, from the previous discussion $\eta_j$ is the time devoted by individual $j$ to studying, which is zero if $j = L$ and $\eta$ otherwise. In turn, the expected wage is determined by the following expression:

$$w_{j,t} \quad \equiv \quad \begin{cases} w_{L,t} & j = L \\ l_{H,A,t}^S w_{H,A,t} + l_{H,Y,t}^S w_{H,Y,t} & j = H \end{cases}, \tag{5}$$

where $l_{H,A,t}^S \equiv L_{H,A,t}^S / L_{H,t}^S$ and $l_{H,Y,t}^S \equiv L_{H,Y,t}^S / L_{H,t}^S$ are the share of high-skilled workings supplied to R&D and manufacturing, respectively, $l_{H,Y,t}^S + l_{H,A,t}^S = 1$. In turn, these wages can be allocated either to consumption in that period, $c_{j,t}$, savings, $s_{j,t}$, or holding real monetary balances, $m_{j,t} \equiv M_t / P_t$, where

$M_t$ are monetary balances and $P_t$ is the aggregate price level. This implies the following flow restriction

$$c_{j,t} + M_t/P_t + s_{j,t} = w_{j,t}\left(1 - \eta_j\right), \tag{6}$$

In period $t+1$, individual $j$ is now a retired old individual which consumes, $d_{j,t+1}$, all of his sources of income, which are real monetary balances held as an adult, $M_{j,t}/P_{j,t+1} = m_{j,t}\left(1 + \pi_{t+1}\right)^{-1}$, where $\pi_{t+1} \equiv P_t/P_{t-1} - 1$ is the inflation rate, the savings made as an adult, $s_{j,t}$, and the corresponding real interest, $r_{t+1}s_{j,t}$, where $r_{t+1}$ is the real interest rate, which is exogenous as a resulting of assuming a small economy with perfect capital mobility with the rest of the world. This implies the following flow restriction

$$d_{j,t+1} = \bar{R}s_{j,t} + M_{j,t}/P_{j,t+1}, \tag{7}$$

where $\bar{R} \equiv (1 + r_{t+1})$. Finally, the relevance of real monetary balances in the previous flow restrictions is determined by assuming, following Hahn and Solow (1995), that a fraction $0 < \mu_j < 1$ of consumption during old age must be financed with real monetary balances. We also assume that lower-skilled individuals due to having a lower ability level than high-skilled individuals face more difficulties in understanding payment technologies that require real money assets, therefore, that $\mu_L > \mu_H$.

We assume that this restriction is binding, implying the following CIA constraint:

$$m_{j,t} = \left(1 + \pi_{t+1}\right)\mu_j d_{j,t+1}, \tag{8}$$

From here, it becomes apparent that inflation by devaluing money forces households to increase their money balances holdings in period 1. Since the real monetary balances held by old individuals are those that they accumulated as adults, that the demand for real money balances in each period $t$, $m_t$, is the sum of the demand of the demand made by young individuals, which we assume to be totally satisfied by the monetary authorities, i.e.

$$m_t^S = m_t^D \equiv \sum_j L_{j,t}^S m_{j,t}, \tag{9}$$

In turn we assume that the government determines the growth rate of the supply of real money balances, $g_{M^S,t+1} \equiv m_{t+1}^S/m_t^S - 1$, according to a chosen inflation target, $\overline{\pi}$, i.e.

$$g_{M^S,t+1} = \left(1 + \overline{\pi}\right)\left(1 + \frac{\dot{m}_t^D}{m_t^D}\right) - 1. \tag{10}$$

If we maximize (3) subject to (6), (7) and (8) we obtain following expressions for optimal consumption in each period

$$c_{j,t} = \frac{w_{j,t}\left(1 - \eta_j\right)}{\beta + 1},\ d_{j,t+1} = \frac{\bar{R}\beta w_{j,t}\left(1 - \eta_j\right)}{\left(\beta + 1\right)\mathcal{P}_j\left(\pi_{t+1}\right)}, \tag{11}$$

where $\mathcal{P}_j\left(\pi_{t+1}\right) \equiv \bar{R}\mu_j\left(\pi_{t+1} + 1\right) - \mu_j + 1$ is a function that incorporates the effects of inflation on consumption which is equal to 1 in the baseline specification without CIA constraints. Replacing (11) in (3), we can obtain the expression for indirect utility and comparing utility levels of individuals of type $j$, we obtain the following condition for individuals to choose to be high-educated:

$$u_{H,t} \geq u_{L,t} \Leftrightarrow \omega(a) \leq \underline{\omega} = \log\left(\mathcal{P}_{L,H}^{\beta}\left(W_{H,L,t}\left(1-\eta\right)\right)^{\beta+1}\right), \tag{12}$$

where $\mathcal{P}_{L,H} \equiv \mathcal{P}_L/\mathcal{P}_H$ and $W_{H,L,t} \equiv W_{H,t}/W_{L,t}$. Considering (4) in (12), and solving for $a$ we can obtain the threshold ability level

$$\omega(a) \leq \underline{\omega} \Leftrightarrow a \geq \underline{a} \equiv \underline{a} = a_{min} + \aleph e^{-\frac{\underline{\omega}}{\theta}} = a_{min} + \aleph\left(\mathcal{P}_{L,H}\left(\pi_{t+1}\right)^{\beta}\left(W_{H,L,t}\left(1-\eta\right)\right)^{\beta+1}\right)^{-\frac{1}{\theta}}. \tag{13}$$

Therefore, considering (13) and (2), we obtain the following expression for the supply of high-skilled workers:

$$l_{H,t}^{S} \equiv 1 - \underline{a} = 1 - a_{min} - \aleph\left(\left(W_{H,L,t}\left(1-\eta\right)\right)^{-\beta-1}\mathcal{P}_{L,H}^{-\beta}\left(\pi_{t+1}\right)\right)^{\frac{1}{\theta}}, \tag{14}$$

From this expression, we derive the following Lemma.

**Lemma 1.** *The threshold level of ability decreases with the skill premium and, in the presence of cash-in-advance constraints in consumption that are more pernicious to low-skilled workers, also inflation. This in turn increases the labor supply.*

*Proof.* If we differentiate $\underline{a}$ with respect to $\pi_{t+1}$ and $W_{H,L,t}$, considering equation (13), we obtain the following expressions, respectively:

$$\frac{\partial \underline{a}}{\partial \pi_{t+1}} = -\frac{\bar{R}\beta\left(\underline{a} - a_{min}\right)\left(\mu_L - \mu_H\right)}{\mathcal{P}_H^2\theta\mathcal{P}_{L,H}}, \quad \frac{\partial \underline{a}}{\partial W_{H,L,t}} = -\frac{\left(\underline{a} - a_{min}\right)\left(\beta+1\right)}{W_{H,L,t}\theta} \tag{15}$$

If is clear that $\frac{\partial \underline{a}}{\partial W_{H,L,t}} < 0$ and, if $\mu_L > \mu_H > 0$, also that $\frac{\partial \underline{a}}{\partial \pi_{t+1}} < 0$. In turn, if we differentiate $l_{H,t}^{S}$ considering (14), we obtain the following expression

$$\frac{\partial l_{H,t}^{S}}{\partial \pi_{t+1}} = -\frac{\partial \underline{a}}{\partial \pi_{t+1}}, \quad \frac{\partial l_{H,t}^{S}}{\partial W_{H,L,t}} = -\frac{\partial \underline{a}}{\partial W_{H,L,t}}.$$

Therefore, $\frac{\partial l_{H,t}^{S}}{\partial \pi_{t+1}} > 0$ and $\frac{\partial l_{H,t}^{S}}{\partial W_{H,L,t}} > 0$. □

We now provide some intuition for this lemma. If low-skilled individuals are more cash-constrained than high-skilled individuals, an increase in inflation has a relative higher impact on the resources available each period for consumption and, therefore, a stronger negative impact on consumption possibilities. An increase of the skill premium has qualitatively the same impact. Both lead to an increase of the incentives for pursuing a college degree, which is reflected on a lower threshold level through (13), as more previously low-skilled workers are willing to endure a higher level of disutility from studying since they would be worse off in the alternative scenario. In turn the assumption that low-skilled individuals are more cash-constrained than high-skilled individuals can be justified by less financial literacy associated with less human capital yielding less holding of non-liquid money, and more precautionary behavior from low skilled families.[2]

[2] Shaw (1996), e.g. show that more-educated individuals are also more likely to be risk takers, thus being less cash constraint. Sequeira (2021) also assumes that as some education expenditures can only be supported with money holdings, then human capital is negatively affected by cash-constraints.

### 3.1.2 Technology

We consider that the aggregate final output, $Y_t$, is produced through the production function below, which uses skilled manufacturers, i.e., high-skilled labor devoted specifically to production, $L_{H,Y,t}$, low-skilled labor, $L_{L,t}$, and different varieties of physical capital in the form of machines and robot, with the quantity employed of variety $z$ being $x_{z,t}$ and the stock of specific blueprints available of different varieties of machines being $A_t$.

$$Y_t = L_{H,Y,t}^{1-\alpha} \left( \sum_{z}^{A_t} x_{z,t}^{\alpha} + L_{L,t}^{\alpha} \right), \tag{16}$$

where $\alpha \in (0,1)$ denotes the elasticity of output with respect to human labor that can be easily automated. We conceptualize technological progress as an increase in the variety of machines in the production process $A_t$, i.e., a growing technological frontier (or TFP growth). Therefore, $A_t$ also represents the technological frontier of the country under consideration at time $t$. Profits of the producer of $Y$ are given by $\Pi_{Y,t} = Y_t - \sum_z^{A_t} p_{z,x,t} x_{z,t} - L_{H,Y,t} w_{H,Y,t} - L_{L,t} w_{L,t}$, where $p_{z,x,t}$, $w_{H,Y,t}$ and $w_{L,t}$ are the prices of machine of type $z$, the wage of high-skilled workers devoted to production and the wage of low-skilled workers, respectively, at time $t$. Considering (16) in the maximization of profits with respect to production factors, yields the following inverse demand expressions for each factor:

$$w_{H,Y,t} = \frac{Y_t (1-\alpha)}{L_{H,Y,t}},\ w_{L,t} = \alpha \left( \frac{L_{H,Y,t}}{L_{L,t}} \right)^{1-\alpha},\ p_{x,z,t} = \alpha \left( \frac{L_{H,Y,t}}{x_{z,t}} \right)^{1-\alpha}, \tag{17}$$

The production of each machine of type $z$ requires a unit of physical capital, which has a cost equal to the gross real interest rate $\overline{R}$, and therefore implies the following expression for profits $\Pi_{z,t} = \left(p_{x,z,t} - \bar{R}\right) x_{z,t}$. It also requires a patented design which is acquired from the R&D sector, which is assumed to only be valid during period $t$, which is consistent with the actual patent length duration of one generation (approximately 20 years) as explained in Prettner and Strunik (2020).

Therefore, we have two types of varieties and, therefore, firms each period. The first one is type $i$ corresponds to newly invented varieties, of which there are $\dot{A}_t$, each protected by a patent, and, therefore, produced by a single producer in monopoly, which implies the following optimal prices, quantities, and profits respectively:

$$p_{i,t} = \frac{\bar{R}}{\alpha},\ x_{i,t} = \left( \frac{\alpha^2}{\bar{R}} \right)^{\frac{1}{1-\alpha}} L_{H,Y,t},\ \Pi_{i,t} = L_{H,Y,t} \bar{R}^{-\frac{\alpha}{1-\alpha}} \alpha^{\frac{\alpha+1}{1-\alpha}} (1-\alpha). \tag{18}$$

The second one is type $j$, corresponds to a old variety, of which there are $A_{t-1}$ quantity, i.e., a variety whose design was invented before period $t$ and, therefore, is no longer protected by a patent. Therefore each of these varieties is produced in perfect competition, implying the following prices, quantities, and profits respectively:

$$p_{j,t} = \bar{R},\ x_{j,t} = \left( \frac{\alpha}{\bar{R}} \right)^{\frac{1}{1-\alpha}} L_{H,Y,t},\ \Pi_{j,t} = 0. \tag{19}$$

Finally, in the R&D sector, new blueprints are produced using scientists, i.e., high-skilled labor devoted to R&D, $L_{H,A,t}$, according to the following standard R&D production

$$\dot{A}_t \equiv A_t - A_{t-1} = L_{H,A,t} \bar{\bar{\delta}}, \tag{20}$$

where $\bar{\bar{\delta}}$ is a parameter that measure the productivity of scientists. Following the standard approach in the literature (Jones, 1995), we consider that the productivity level itself depends on a baseline exogenous level, $\delta$, affected by: (i) intertemporal knowledge spillovers (the standing-on-giants'-shoulders externality), and (ii) the duplication effects (the "stepping-on-toes" externality). These strength of these effects is measured, respectively, $0 < \varphi < 1$ and $0 < 1 - \lambda < 1$, which is reflected in the following standard specification:

$$\bar{\bar{\delta}} = \delta A_{t-1}^{\varphi} / L_{H,A,t}^{1-\lambda}. \tag{21}$$

The expected profits of a firm devoting resources to invent a new machine blueprint are $\Pi_{A,i} = p_{A,i} - L_{i,H,A,t} w_{H,A,t}$, where $p_{A,i}$ is the price of a new patent and $L_{i,H,A,t} \equiv L_{H,A,t}/\dot{A}_t$ is the quantity of scientists employed by a single firm. Assuming free entry and considering that $p_{A,i} = \Pi_{i,t}$ as well as (20) and (21) implies the following optimal wage for scientists:

$$w_{H,A,t} = \frac{\dot{A}_t}{L_{H,A,t}} p_{A,i} = \delta A_{t-1}^{\varphi} L_{H,A,t}^{\lambda-1} \Pi_{i,t}. \tag{22}$$

### 3.1.3 General equilibrium expressions and analytical results

We now characterize the general equilibrium of the economy, where all individuals maximize utility, firms maximize profits and goods market and labor markets are in equilibrium.

Replacing the expression for optimal profits of the producer of variety $i$ in (18) in the optimal wage for scientists in (22), and making use of (2) to express labor quantities as portions, we obtain the following GE wage for scientists:

$$w_{H,A,t} = A_{t-1}^{\varphi} \mathcal{C}_1 l_{H,A,t}^{\lambda-1} l_{H,Y,t} \left(L_t l_{H,t}\right)^{\lambda}, \tag{23}$$

where $\mathcal{C}_1 = \bar{R}^{-\frac{\alpha}{1-\alpha}} \delta \alpha^{\frac{\alpha+1}{1-\alpha}} (1-\alpha)$. From this expression it is clear that the wage of scientists depend (i) negatively on the share of scientists, due to the duplication effect, (ii) positively on the share of skilled manufacturers since they have a positive impact on demand and, hence, profits of machine producers, leading to more incentives to allocate resources to R&D and, therefore, increasing the productivity of scientists, positively on the share of skilled workers due to the combination of previous effects and the fact that (ii) more than compensates for (i).

Considering that $\sum_z^{A_t} x_{z,t}^{\alpha} = A_{t-1} x_{j,t}^{\alpha} + \dot{A}_t x_{i,t}^{\alpha}$, the R&D production (20) and 21, and the he expressions for optimal number machines of each variety in (18) and (19)we obtain the following expression for aggregate output:

$$Y_t = L_{H,Y,t} \left(A_{t-1} \mathcal{C}_3 + A_{t-1}^{\varphi} L_{H,A,t}^{\lambda} \mathcal{C}_2\right) + L_{H,Y,t}^{1-\alpha} L_{L,t}^{\alpha}, \tag{24}$$

where $\mathcal{C}_2 \equiv \mathcal{C}_2 = \delta \left(\alpha^2/\bar{R}\right)^{\frac{\alpha}{1-\alpha}}$ and $\mathcal{C}_3 = \left(\alpha/\bar{R}\right)^{\frac{\alpha}{1-\alpha}}$. Replacing (24) in the expression for wages in (17) and considering once again (2), we obtain the following general equilibrium expression for wages for high-skilled workers in the manufacturing sector:

$$w_{H,Y,t} = (1-\alpha) \left( A_{t-1} \mathcal{C}_3 + A_{t-1}^{\varphi} \mathcal{C}_2 \left(L_t l_{H,A,t} l_{H,t}\right)^{\lambda} + \left(\frac{l_{L,t}}{l_{H,Y,t} l_{H,t}}\right)^{\alpha} \right). \tag{25}$$

From this expression it is clear that the wage of skilled manufacturers (i) depends positively on the share of low-skilled workers, (ii) negatively on the share of skilled manufacturers, due to decreasing returns, (iii) positively on the share of scientists due to their contribution to creating varieties of machines used in production and, therefore, increasing their productivity, and (iv) positively or negatively on the share of skilled workers according to which effect, (ii) or (iii), dominates, which, in turn, depends on the stock of knowledge of the economy.

Considering (17), we obtain the following expression for the wage of low-skilled workers:

$$w_{L,t} = \alpha \left( \frac{l_{H,Y,t} l_{H,t}}{l_{L,t}} \right)^{1-\alpha}. \tag{26}$$

From this expression it is clear that the wage of low-skilled workers depends (i) negatively on the share of low-skilled workers, (ii) positively on the share of skilled manufacturers, (iii) positively on the share of skilled workers.

Dividing (25), (23) (26) across each other, expressing all in terms of shares of $l_{H,A,t}$ and $l_{H,t}$, and further simplifying, we obtain expressions for the wage differentials between different types of workers:

$$W_{H,Y,A,t} \equiv \frac{w_{H,Y,t}}{w_{H,A,t}} = \frac{l_{H,A,t}}{1-l_{H,A,t}} \left( \mathcal{C}_4 + \left( \frac{\mathcal{F}_{1,t} + \mathcal{F}_{2,t} \left( \frac{1-l_{H,t}}{l_{H,t}(1-l_{H,A,t})} \right)^{\alpha}}{l_{H,A,t} l_{H,t}^{\lambda}} \right) \right), \tag{27}$$

$$W_{H,A,L,t} \equiv \frac{w_{H,A,t}}{w_{H,L,t}} = \mathcal{F}_{3,t} l_{H,A,t}^{\lambda-1} l_{H,t}^{\lambda+\alpha-1} \left(1-l_{H,A,t}\right)^{\alpha} \left(1-l_{H,t}\right)^{1-\alpha}, \tag{28}$$

$$W_{H,Y,L,t} \equiv \frac{w_{H,Y,t}}{w_{L,t}} = \mathcal{F}_{3,t} l_{H,A,t}^{\lambda} l_{H,t}^{\lambda+\alpha-1} \left( \frac{1-l_{H,t}}{1-l_{H,A,t}} \right)^{1-\alpha} \left( \begin{array}{c} \mathcal{C}_4 + \left( \frac{1}{l_{H,A,t} l_{H,t}} \right)^{\lambda} \times \\ \times \left( \mathcal{F}_{1,t} + \mathcal{F}_{2,t} \left( \frac{1-l_{H,t}}{l_{H,t}(1-l_{H,A,t})} \right)^{\alpha} \right) \end{array} \right), \tag{29}$$

where $\mathcal{F}_{1,t} = \frac{A_{t-1}^{1-\varphi} L_t^{-\lambda} \mathcal{C}_3}{\mathcal{C}_1}$, $\mathcal{F}_{2,t} = \frac{A_{t-1}^{-\varphi} L_t^{-\lambda}}{\mathcal{C}_1}$, $\mathcal{F}_{3,t} \equiv \frac{A_{t-1}^{\varphi} L_t^{\lambda} \mathcal{C}_1 (1-\alpha)}{\alpha}$ and $\mathcal{C}_4 = \frac{\mathcal{C}_2 \delta}{\mathcal{C}_1}$. From here we derive the following lemma:

**Lemma 2.** *In the presence of cash-in-advance constraints in consumption that are more pernicious to low-skilled workers (i) an increase of the share of scientists increases the wage of skilled manufacturers relative to other workers and decreases the wage of scientists relative to low-skilled workers, (ii) an increase of the share of skilled workers decreases the manufacturing wage premium and, for sufficiently high share of skilled workers, also the wage of scientists and skilled manufacturers relative to low-skilled workers.*

*Proof.* Taking the derivatives of (27), (28) and (29) with respect to $l_{H,t}$ results in the following expressions, respectively, with the signals indicated below each one:

$$\underbrace{\frac{\partial W_{H,Y,A,t}}{\partial l_{H,A,t}}}_{+} = \frac{\mathcal{C}_4 + \left(\frac{1}{l_{H,A,t}l_{H,t}}\right)^{\lambda}\left(\mathcal{F}_{1,t}\left(1-\lambda l_{H,Y,t}\right) + \mathcal{F}_{2,t}\left(\frac{l_{L,t}}{l_{H,Y,t}l_{H,t}}\right)^{\alpha}\left(1-\lambda l_{H,Y,t}+\alpha l_{H,A,t}\right)\right)}{l_{H,Y,t}^{2}},$$

$$\underbrace{\frac{\partial W_{H,A,L,t}}{\partial l_{H,A,t}}}_{-} = -\mathcal{F}_{3,t} l_{H,A,t}^{\lambda-2} l_{H,Y,t}^{\alpha-1} l_{H,t}^{\lambda+\alpha-1} l_{L,t}^{1-\alpha}\left(\alpha l_{H,A,t} + l_{H,Y,t}\left(1-\lambda\right)\right),$$

$$\underbrace{\frac{\partial W_{H,Y,L,t}}{\partial l_{H,A,t}}}_{+} = \mathcal{F}_{3,t} l_{H,A,t}^{\lambda-1} l_{H,Y,t}^{\alpha-2} l_{H,t}^{\lambda+\alpha-1} l_{L,t}^{1-\alpha}\left(\begin{array}{c}\mathcal{C}_4\left(\lambda l_{H,Y,t}+l_{H,A,t}\left(1-\alpha\right)\right)+\\ +l_{H,A,t}^{1-\lambda} l_{H,t}^{-\lambda}\left(\mathcal{F}_{1,t}\left(1-\alpha\right)+\mathcal{F}_{2,t}\left(\frac{l_{L,t}}{l_{H,Y,t}l_{H,t}}\right)^{\alpha}\right)\end{array}\right).$$

Taking the derivatives of such equations with respect to $l_{H,A,t}$ results in the following expressions, respectively, with the signals indicated below each one:

$$\underbrace{\frac{\partial W_{H,Y,A,t}}{\partial l_{H,t}}}_{-} = -\frac{\lambda \mathcal{F}_{1,t} l_{L,t} + \mathcal{F}_{2,t}\left(\frac{l_{L,t}}{l_{H,Y,t}l_{H,t}}\right)^{\alpha}\left(\lambda l_{L,t}+\alpha\right)}{l_{H,A,t}^{\lambda-1} l_{H,Y,t} l_{L,t} l_{H,t}^{1+\lambda}},$$

$$\underbrace{\frac{\partial W_{H,A,L,t}}{\partial l_{H,t}}}_{+/-} = -\lambda \mathcal{F}_{3,t} l_{H,A,t}^{\lambda-1} l_{H,Y,t}^{\alpha} l_{H,t}^{\lambda+\alpha-2} l_{L,t}^{-\alpha}\left(l_{H,t}-\tilde{l}_H\right)\begin{cases} >0 & \text{if } l_{H,t} > \tilde{l}_H \\ <0 & \text{otherwise}\end{cases},$$

$$\underbrace{\frac{\partial W_{H,Y,L,t}}{\partial l_{H,t}}}_{+/-} = \begin{array}{c} -\mathcal{F}_{3,t} l_{H,A,t}^{\lambda} l_{H,Y,t}^{\alpha-1} l_{H,t}^{\lambda+\alpha-2} l_{L,t}^{-\alpha}\times \\ \times\left(\begin{array}{c}\mathcal{C}_4\left(l_{H,t}-\tilde{l}_H\right)+\left(\frac{1}{l_{H,A,t}l_{H,t}}\right)^{\lambda}\times\\ \times\left(\mathcal{F}_{1,t}\left(1-\alpha\right)+\mathcal{F}_{2,t}\left(\frac{l_{L,t}}{l_{H,Y,t}l_{H,t}}\right)^{\alpha}\right)\end{array}\right)\end{array}\begin{cases} <0 & \text{if } l_{H,t} > \tilde{l}_H \\ \gtreqless 0 & \text{otherwise}\end{cases},$$

where $\tilde{l}_H = 1 - \frac{1-\alpha}{\lambda}$. □

We now provide intuition for this lemma, which is summarized in Table 6. An increase of the share of scientists implies an (i) increase of the number of scientists, and (ii) a reduction of the number of skilled manufacturers. Effect (i) implies a (i-a) reduction of the productivity of scientists, (i-b) an increase of the number of varieties, while effect (ii) leads to a (ii-a) direct positive impact on the productivity of skilled manufacturers due to decreasing returns in production, (ii-b) a direct negative impact on the productivity of low-skilled workers also due to decreasing returns and (ii-c) a decrease of the demand of machines, decreasing profits and incentives to allocate resources to R&D and, therefore, leading to a fall of productivity and wages of scientists. All these effects imply that the wages of skilled manufacturers increase relative to all other workers, i.e., scientists and low-skilled workers.

An increase of the share of skilled workers has different impacts because, while it leads to an (i) increase of the number of scientists, an, in turn, effects (i-a) and (i-b), it also (iii) increases the number of skilled workers and (iv) reduces the number of low-skilled workers. The combination of effect (iii) and (iv) causes an increase of the wage of low-skilled workers and also has (iii) a negative impact on productivity and wage of skilled manufacturers (iii-b) and a positive impact on the wage of scientists (iii-c). These effects have an ambiguous impact on the wage of skilled manufacturers and increase the wages of the remaining workers, i.e., scientists and low-skilled workers.

In general equilibrium, (i) the distribution of skilled workers across sectors is such that wages of scientists and skilled manufacturers are equal and (ii) the labor market is in equilibrium. In turn, condition (i) implies that (i-a) the share of skilled workers adjusts endogenously to changes in the other variables and parameters such that wages of skilled manufacturers and scientists are all always equal (i-b) the skill premium is equal to the wage of scientists relative to low-skilled labor, which, in turn, is equal to the manufacturing skill premium. These implications are represented, respectively, by the following equations:

$$l_{H,A,t} = l^A_{H,A,t}(l_{H,t}) : W^*_{H,Y,A,t}\left(l_{H,t}, l^A_{H,A,t}(l_{H,t})\right) = 1, \tag{30}$$
$$W^A_{H,L,t} = W^A_{H,L}\left(l_{H,t}\right) = W_{H,A,L,t}\left(l_{H,t}, l^A_{H,A,t}(l_{H,t})\right) = W_{H,Y,L,t}\left(l_{H,t}, l^A_{H,A,t}(l_{H,t})\right). \tag{31}$$

From here, we define the following lemma, which will be important later on.

**Lemma 3.** *In the presence of cash-in-advance constraints in consumption that are more pernicious to low-skilled workers, the satisfaction of the arbitrage condition implies that (i) the share of scientists increases with the share of skilled workers, and (ii) the skill premium decreases with the share of skilled workers, for a sufficiently high share of skilled workers.*

*Proof.* Applying the implicit function theorem to (30), yields the following derivative, with the signal indicated below:

$$\underbrace{\frac{\partial l^A_{H,A,t}}{\partial l_{H,t}}}_{+} = -\underbrace{\frac{\partial\, W_{H,Y,A,t}}{\partial\, l_{H,t}}}_{-}\left(\underbrace{\frac{\partial\, W_{H,Y,A,t}}{\partial\, l_{H,A,t}}}_{+}\right)^{-1}. \tag{32}$$

Since by equation (31), $W^A_{H,L,t} = W_{H,A,L,t}\left(l_{H,t}, l^A_{H,A,t}(l_{H,t})\right)$, if we differentiate this expression with respect to $l_{H,t}$ we obtain the following expression, with the signal indicated below if $l_H > \tilde{l}_H$:

$$\underbrace{\frac{\partial W^A_{H,L}}{\partial l_{H,t}}}_{-} = \underbrace{\frac{dW_{H,L,t}}{dl_{H,t}}}_{-} = \underbrace{\frac{\partial\, W_{H,A,L,t}}{\partial\, l_{H,A,t}}}_{-}\underbrace{\frac{\partial l^A_{H,A,t}}{\partial l_{H,t}}}_{+} + \underbrace{\frac{\partial\, W_{H,A,L,t}}{\partial\, l_{H,t}}}_{-}. \tag{33}$$

□

Condition (ii) implies that $l^*_{H,t} = l_{H,t} = l^S_{H,t}$. Considering this as well as the equation derived from condition (i), we obtain the following system of two equations, which can be solved implicitly for the general equilibrium quantities of skilled labor and scientists:

$$l^*_{H,t} = 1 - a_{min} - \aleph\left(\mathcal{P}_{L,H}\left(\pi_{t+1}\right)^{\beta}\left(\left(1-\eta\right)W^A_{H,L}\left(l^*_{H,t}\right)\right)^{\beta+1}\right)^{-\frac{1}{\theta}}, \tag{34}$$
$$l^*_{H,A,t} = l^A_{H,A,t}(l_{H,t}) \Leftrightarrow W^*_{H,Y,A,t}\left(l_{H,t}, l^A_{H,A,t}(l_{H,t})\right) = 1. \tag{35}$$

From here we derive the following proposition concerning the impacts of inflation on the Skill Premium and workers distribution.

**Proposition 1.** *In the presence of cash-in-advance constraints in consumption that are more pernicious to low-skilled workers, for a sufficiently high portion of skilled workers an increase of expected inflation has a contemporaneous positive effect on the portion of high-skilled workers and scientists and a negative impact on the Skill Premium.*

*Proof.* Applying the implicit function theorem to (30), yields the following derivative, with the signal indicated below if $l_H > \tilde{l}_H$:

$$\underbrace{\frac{\partial\, l^*_{H,t}}{\partial\, \pi_{t+1}}}_{+} = - \underbrace{\frac{\partial\, \underline{a}}{\partial\, \pi_{t+1}}}_{-} \left( \underbrace{\frac{\partial\, \underline{a}}{\partial\, W_{H,L}}}_{-} \underbrace{\frac{d\, W^A_{H,L,t}}{dl_{H,t}}}_{-} +1 \right)^{-1} . \tag{36}$$

In turn, considering that $l^*_H = l^*_H(\pi_{t+1})$ in (35) and the skill premium in (31) leads to the following derivatives, with the signals indicated below if $l_H > \tilde{l}_H$:

$$\underbrace{\frac{\partial\, l^*_{H,A,t}}{\partial\, \pi_{t+1}}}_{+} = \underbrace{\frac{\partial l^A_{H,A,t}}{l_{H,t}}}_{+} \underbrace{\frac{\partial\, l^*_{H,t}}{\partial\, \pi_{t+1}}}_{+}, \underbrace{\frac{\partial\, W^*_{H,L,t}}{\partial\, \pi_{t+1}}}_{-} = \underbrace{\frac{d\, W^A_{H,L,t}}{dl_{H,t}}}_{-} \underbrace{\frac{\partial\, l^*_{H,t}}{\partial\, \pi_{t+1}}}_{+} .$$

□

The intuition is as follows. An increase in expected inflation leads to an increase of supply of skilled workers as explained in lemma 1. This, as explained in lemmas 2 and 3, increases the wages of scientists due to the additional demand by new skilled manufacturers that more than compensates the duplication effect. The impacts on the wage of skilled manufacturers are ambiguous, depending on the level of stock of knowledge of the economy, but are always inferior in magnitude to the previous effects. Therefore, the relative wage of scientists increase and, consequently, the portion of scientists also increase as there are more incentives for skilled workers to choose to work in the R&D sector rather than the manufacturing sector. Concurrently, the wage of low-skilled workers increases due to their higher productivity, which more than compensates any change in the wage of skilled workers, leading to a decrease of the skill premium.[3]

Finally, we consider the following defining expression for the labor share in the manufacturing sector ($lsm_t$) as the sum of the labor in that sector of low- and high-skilled individuals ($lsm_{L,t}$ and $lsm_{H,t}$, respectively):

$$lsm_t \equiv lsm_{L,t} + lsm_{H,t},\, lsm_L \equiv \frac{L_{L,t} w_{L,t}}{Y_t},\, lsm_H \equiv \frac{L_{H,Y,t} w_{H,Y,t}}{Y_t}. \tag{37}$$

Considering once again the expressions for general equilibrium wages of high-skilled workers (25), the general equilibrium wages of low skilled workers (17), and general equilibrium output (24), we can obtain

[3] The causes a decrease of the supply of skilled workers, which attenuates the previous one but does not cancel them out.

the following expressions for the manufacturing labor share of low- and high-skilled respectively:

$$lsm^*_{L,t} = \frac{\alpha}{A_{t-1}\mathcal{C}_2\left(\frac{(1-l^*_{H,A,t})l^*_{H,t}}{1-l^*_{H,t}}\right)^{\alpha}\left(\alpha^{-\frac{\alpha}{1-\alpha}} + A^{\varphi-1}_{t-1}\delta\left(L_t l^*_{H,A,t} l^*_{H,t}\right)^{\lambda}\right)+1},\; lsm^*_{H,t} \equiv 1-\alpha. \tag{38}$$

From here we can derive the following proposition concerning how the labor share is affected by inflation.

**Proposition 2.** *In the presence of cash-in-advance constraints in consumption that are more pernicious to low-skilled workers, under certain conditions, in the short and medium-run an increase of expected inflation decreases he manufacturing labor share with ambiguous effects otherwise. In the long-run it does not has any effect.*

*Proof.* If we differentiate (38) with respect to inflation by applying the chain rule, we obtain the following derivative of the labor share valid for the short- and medium-run, with the the signal indicated if $l_{H,t} > \tilde{l}_H$ considering proposition 1:

$$\underbrace{\frac{\partial\, lsm^*_t}{\partial\, \pi_{t+1}}}_{+/-} = \underbrace{\frac{\partial\, lsm^*_{L,t}}{\partial\, l^*_{H,t}}}_{-}\underbrace{\frac{\partial\, l^*_{H,t}}{\partial\, \pi_{t+1}}}_{+/-} + \underbrace{\frac{\partial\, lsm^*_{L,t}}{\partial\, l^*_{H,A,t}}}_{+/-}\underbrace{\frac{\partial\, l^*_{H,A,t}}{\partial\, \pi_{t+1}}}_{+}\begin{cases} <0 & \text{if } \left\{\begin{array}{c} l_{H,t} > \tilde{l}_H \wedge \\ \wedge\, (\lambda l_{H,Y,t} - \alpha l_{H,A,t}) > \\ > \frac{\mathcal{C}_3\alpha l_{H,A,t}}{A^{\varphi-1}_{t-1}L^{\lambda}_t\mathcal{C}_2 l^{\lambda}_{H,A,t} l^{\lambda}_{H,t}} \end{array}\right\} \\ \gtreqless 0 & \text{otherwise} \end{cases}. \tag{39}$$

where

$$\underbrace{\frac{\partial\, lsm^*_{L,t}}{\partial\, l^*_{H,t}}}_{-} = -\mathcal{G}_t l_{H,A,t} l_{H,Y,t}\left(A^{\varphi-1}_{t-1}L^{\lambda}_t\mathcal{C}_2 l^{\lambda}_{H,A,t} l^{\lambda}_{H,t}(\lambda l_{L,t}+\alpha) + \mathcal{C}_3\alpha\right),$$

$$\underbrace{\frac{\partial\, lsm^*_{L,t}}{\partial\, l^*_{H,A,t}}}_{+/-} = -\mathcal{G}_t l_{H,t} l_{L,t}\left(\underbrace{\begin{array}{c} A^{\varphi-1}_{t-1}L^{\lambda}_t\mathcal{C}_2 l^{\lambda}_{H,A,t} l^{\lambda}_{H,t}\times \\ (\lambda l_{H,Y,t} - \alpha l_{H,A,t}(1+\mathcal{C}_3)) \end{array}}_{+/-}\right)\begin{cases} <0 & \text{if } \left\{\begin{array}{c} \lambda l_{H,Y,t} - \alpha l_{H,A,t} > \\ > \frac{\mathcal{C}_3\alpha l_{H,A,t}}{A^{\varphi-1}_{t-1}L^{\lambda}_t\mathcal{C}_2 l^{\lambda}_{H,A,t} l^{\lambda}_{H,t}} \end{array}\right\} \\ \gtreqless 0 & \text{otherwise} \end{cases},$$

$$\mathcal{G}_t \equiv \frac{A_{t-1}\alpha}{l_{H,A,t} l^{1-\alpha}_{H,Y,t} l^{1-\alpha}_{H,t} l^{1+\alpha}_{L,t}\left(A_{t-1} l^{\alpha}_{H,Y,t} l^{\alpha}_{H,t} l^{-\alpha}_{L,t}\left(A^{\varphi-1}_{t-1}L^{\lambda}_t\mathcal{C}_2 l^{\lambda}_{H,A,t} l^{\lambda}_{H,t} + \mathcal{C}_3\right)+1\right)^2}.$$

In the long-run, $A_{t-1}$ tends to infinity and, therefore, $\lim_{A_{t-1}\to\infty}\frac{\partial\, lsm^*_t}{\partial\, \pi_{t+1}} = 0$. □

The intuition is as follows. An increase of the share of skilled workers increases the marginal productivity of low-skilled workers, which increases both wages and output per low-skilled worker, but also increases the number of varieties, which increases output per low-skilled worker but does not affect its marginal productivity. Therefore, output per capita is more positively affected than wages, leading to a fall of the labor share through this channel. On the other hand, an increase of the share of scientists increases the number of varieties, which increases output per capita, but decreases the productivity of low-skilled workers, which decreases both wages and output per capita. Therefore, there are contradictory impacts on output per capita. Under certain conditions – $\lambda l_{H,Y,t} - \alpha l_{H,A,t} > \frac{\mathcal{C}_3 \alpha l_{H,A,t}}{A_{t-1}^{\varphi-1} L_t^{\lambda} \mathcal{C}_2 l_{H,A,t}^{\lambda} l_{H,t}^{\lambda}} \Longrightarrow \lambda l_{H,Y,t} > \alpha l_{H,A,t} \left(1 + \frac{\mathcal{C}_3}{A_{t-1}^{\varphi-1} L_t^{\lambda} \mathcal{C}_2 l_{H,A,t}^{\lambda} l_{H,t}^{\lambda}}\right)$ – the fall in output per capita is smaller than the fall in wages, leading to a fall in the labor share through this particular channel. Therefore, by leading to an increase of the share of both skilled workers and scientists, inflation will generally decrease the labor share through both channels under the previously mentioned conditions, with ambiguous impacts otherwise.

### 3.2 Cash-in-advance constraints in R&D sector firms

Following the literature (e.g., Chu and Cozzi, 2014, Afonso and Lima, 2022; Afonso and Sequeira, 2023; Gil and Iglesias, 2019), we now assume alternatively the existence of a CIA constraint in the R&D sector by assuming that a fraction $\Omega$ of the R&D costs must be paid using money balances. In particular, we assume that firms borrow $b_{i,t}$ households to pay for a fraction $\Omega$ of wages, $w_{H,A,t} L_{H,A,t}$, implying the following new expression for R&D costs:

$$\text{R\&D costs} = \underbrace{(1-\Omega)\, w_{H,A,t} L_{H,A,t}}_{\substack{\text{wages which may} \\ \text{be paid in credit}}} + (1+\iota) \underbrace{\Omega w_{H,A,t} L_{H,A,t}.}_{\substack{\text{wages which must} \\ \text{be paid in cash}}} \tag{40}$$

Therefore the real money demand by firms is represented by $b_{i,t} = \Omega w_{H,A,t} L_{H,A,t}$. Assuming once again free entry and considering that $p_{A,i} = \Pi_{i,t}$, and simplify (40) we obtain the new free entry condition

$$\dot{A}_t \Pi_{i,t} = (1 + \Omega\iota)\, w_{H,A,t} L_{H,A,t}.$$

From here and considering (20) and (21) we obtain the following expression for the new optimal wage for scientists:

$$w_{H,A,t} = \frac{\dot{A}_t}{L_{H,A,t}} p_{A,i} = \frac{\delta A_{t-1}^{\varphi} L_{H,A,t}^{\lambda-1} \Pi_{i,t}}{1 + \Omega \iota_{t+1}}. \tag{41}$$

The government determines the nominal interest rate according to the Fisher effect, i.e., they set the real interest based on an inflation target previously defined and the real interest rate.

$$\iota_{t+1} = (1 + \pi_{t+1})\, \bar{R} - 1. \tag{42}$$

Therefore, considering (42) in (41), permits establishing a link between wages and inflation through the nominal interest rate. An increase in inflation leads monetary authorities to feel the necessity of increasing interest rates so as to keep the real interest rate in line with the world real interest rate. This effectively increases that R&D firms face, which leads them to decrease their demand for scientists, which is reflected in a lower equilibrium wage than before.

From the households perspective, the supply of money balances is as follows:

$$b_t^S \equiv b_{L,t} L_{L,t} + b_{H,t} L_{H,t}.$$

where $b_{j,t}$ is the cash loan conceded by individual of type $j$, i.e., the real money balances supplied by individual of type $j$, which is assumed to be repaid with interest in the following period. Also following the literature (e.g., Chu and Cozzi, 2014), since we assume no other reason for households to hold real money balances, we assume that $b_{j,t} = m_{j,t}$, i.e., we assume that they accumulate money balances with the sole purpose of conceding loans to firms. Considering this and (42) in the previous expressions of the flow constraints in equations (6) and (7), leads to the new flow restrictions for periods 1 and 2.

$$c_{j,t} + s_{j,t} + b_{j,t} = w_{j,t}\left(1-\eta_j\right),\; d_{j,t+1} = \bar{R}\left(s_{j,t} + b_{j,t}\right). \tag{43}$$

The optimal expressions for consumption in each period are now as follows:

$$c_{j,t} = \frac{w_{j,t}\left(1-\eta_j\right)}{\beta+1},\; d_{j,t+1} = \frac{\bar{R}\beta w_{j,t}\left(1-\eta_j\right)}{\left(1+\beta\right)}, \tag{44}$$

This in turn leads to the following expressions for the threshold level of ability and supply of workers, respectively:

$$\underline{a} = a_{min} + \aleph\left(W_{H,L,t}\left(1-\eta\right)\right)^{-\frac{1+\beta}{\theta}},\; l_{H,t}^{S} = 1 - a_{min} - \aleph\left(W_{H,L,t}\left(1-\eta\right)\right)^{-\frac{1+\beta}{\theta}}. \tag{45}$$

Therefore, the existence of CIA constraints in the R&D sector does not affect households decisions concerning consumption and labor supply. This result from the fact that the households the interest that households receive from savings is the same as the one they earn from the loans conceded to R&D firms, causing them to be indifferent to whether such amount is high or low.

In general equilibrium, the expressions for wages are as follows:

$$w_{H,Y,t} = (1-\alpha)\left(A_{t-1}\mathcal{C}_3 + A_{t-1}^{\varphi}\mathcal{C}_2\left(L_t l_{H,A,t} l_{H,t}\right)^{\lambda} + \left(\frac{l_{L,t}}{l_{H,Y,t} l_{H,t}}\right)^{\alpha}\right) \tag{46}$$

$$w_{H,A,t} = \frac{A_{t-1}^{\varphi}\mathcal{C}_1 l_{H,A,t}^{\lambda-1} l_{H,Y,t}\left(L_t l_{H,t}\right)^{\lambda}(1-\alpha)}{\Omega\iota_{t+1}+1},\; w_{L,t} = \alpha\left(\frac{l_{H,Y,t} l_{H,t}}{l_{L,t}}\right)^{1-\alpha}, \tag{47}$$

where the only that is different the one concerning wages of scientists, which are now directly affected by inflation through the interest rate, by the reasons explained above. From here we obtain the following expressions for wage differentials:

$$W_{H,Y,A,t} = \frac{l_{H,A,t}\left(\mathcal{C}_4 + \left(\frac{1}{l_{H,A,t} l_{H,t}}\right)^{\lambda}\left(\mathcal{F}_{1,t} + \mathcal{F}_{2,t}\left(\frac{1-l_{H,t}}{l_{H,t}(1-l_{H,A,t})}\right)^{\alpha}\right)\right)\left(\Omega\iota_{t+1}+1\right)}{1-l_{H,A,t}}, \tag{48}$$

$$W_{H,A,L,t} = \frac{\mathcal{F}_{3,t} l_{H,A,t}^{\lambda-1} l_{H,t}^{\lambda+\alpha-1}\left(1-l_{H,A,t}\right)^{\alpha}\left(1-l_{H,t}\right)^{1-\alpha}}{\Omega\iota_{t+1}+1}, \tag{49}$$

$$W_{H,Y,L,t} = \mathcal{F}_{3,t} l_{H,A,t}^{\lambda} l_{H,t}^{\lambda+\alpha-1}\left(\frac{1-l_{H,t}}{1-l_{H,A,t}}\right)^{1-\alpha}\left(\begin{array}{c}\mathcal{C}_4 + \left(\frac{1}{l_{H,A,t} l_{H,t}}\right)^{\lambda}\times \\ \times\left(\mathcal{F}_{1,t} + \mathcal{F}_{2,t}\left(\frac{1-l_{H,t}}{l_{H,t}(1-l_{H,A,t})}\right)^{\alpha}\right)\end{array}\right). \tag{50}$$

From here we derive the following lemma:

**Lemma 4.** *In the presence of CIA constraints in the R&D sector, an increase of the share of scientists and skilled workers have the same impacts as described in lemma 2 under the same conditions and (iii) an* *increase in inflation increases the wage of skilled manufacturers relative to scientists, it decreases the wage of scientists relative to low-skilled workers and does not affect the wage of skilled manufacturers relative to low-skilled workers.*

*Proof.* take the derivatives of (48), (49) and (50) with respect to $l_{H,t}$ and $l_{H,A,t}$ and obtain qualitatively similar derivatives as those of lemma 2, with the difference being that some are multiplied or divided by $(\Omega\iota_{t+1}+1)$. Since this term is positive, the signal remains unaffected. In turn, taking the derivatives of these equations with respect to $\pi_{t+1}$, we obtain the following expressions:

$$\underbrace{\frac{\partial\, W_{H,Y,A,t}}{\partial\, \pi_{t+1}}}_{+}=\frac{\Omega\bar{R}W_{H,Y,A,t}}{\Omega\iota_{t+1}+1},\ \underbrace{\frac{\partial\, W_{H,A,L,t}}{\partial\, \pi_{t+1}}}_{-}=-\frac{\Omega\bar{R}W_{H,A,L,t}}{\Omega\iota_{t+1}+1},\ \frac{\partial W_{H,Y,L,t}}{\partial \pi_{t+1}}=0$$

. □

In what concerns the implications of the arbitrage condition they are now represented by the following equations:

$$l_{H,A,t} = l^A_{H,A,t}(l_{H,t},\pi_{t+1}) : W^*_{H,Y,A,t}\left(l_{H,t}, l_{H,A,t}\right)=1, \tag{51}$$

$$W^A_{H,L,t} = W^A_{H,L}\left(l_{H,t}\right)=W_{H,A,L,t}\left(l_{H,t}, l^A_{H,A,t}(l_{H,t},\pi_{t+1}),\pi_{t+1}\right)= \tag{52}$$

$$= W_{H,Y,L,t}\left(l_{H,t}, l^A_{H,A,t}(l_{H,t},\pi_{t+1}),\pi_{t+1}\right). \tag{53}$$

Therefore, the most important change is that as the result of inflation affecting the wage of skilled manufacturers relative to scientists, it now also affects directly the distribution of skilled workers across sectors, leading to lemma 5.

**Lemma 5.** *In the presence of CIA constraints in the R&D sector, the satisfaction of the arbitrage condition implies the same results and conditions of 3 and also that (iii) the share of scientists decreases with inflation, and (iv) the skill premium increases with inflation.*

*Proof.* Since the signal of the derivatives of wage differentials with respect to $l_{H,t}$ and $l_{H,A,t}$ remains the same, it continues to be the case $\frac{\partial l^A_{H,A,t}}{\partial l_{H,t}}>0$ and $\frac{\partial W^A_{H,L}}{\partial l_{H,t}}<0$. In what concerns inflation, the application of the implicit function theorem to (30) yields the following derivative with respect to $\pi_{t+1}$, with the signal indicated below:

$$\underbrace{\frac{\partial l^A_{H,A,t}}{\partial \pi_{t+1}}}_{-}=-\underbrace{\frac{\partial\, W_{H,Y,A,t}}{\partial\, \pi_{t+1}}}_{+}\left(\underbrace{\frac{\partial\, W_{H,Y,A,t}}{\partial\, l_{H,A,t}}}_{+}\right)^{-1}. \tag{54}$$

Since by equation (31), $W^A_{H,L,t}=W_{H,Y,L,t}\left(l_{H,t}, l^A_{H,A,t}(l_{H,t},\pi_{t+1}),\pi_{t+1}\right)$, if we differentiate this expression with respect to $l_{H,t}$ and $\pi_{t+1}$ we obtain the following derivatives, with the signal indicated below:

$$\underbrace{\frac{\partial W^A_{H,L}}{\partial \pi_{t+1}}}_{-} = \underbrace{\frac{dW_{H,Y,L,t}}{d\pi_{t+1}}}_{-} = \underbrace{\frac{\partial\, W_{H,Y,L,t}}{\partial\, l_{H,A,t}}}_{+}\underbrace{\frac{\partial l^A_{H,A,t}}{\partial \pi_{t+1}}}_{-} + \underbrace{\frac{\partial\, W_{H,Y,L,t}}{\partial\, \pi_{t+1}}}_{0}. \tag{55}$$

□

This result is quite intuitive. Since an increase in inflation decreases the demand for scientists and their wages, this creates incentives for skilled workers to move from the R&D sector to the manufacturing sector. This leads to an increase of the wage of low-skilled workers at the same time that decreases the average wage of skilled workers, which implies a decrease of the skill premium.

Considering both the arbitrage condition and the labor market condition leads to the following new system of equations that the determine the GE quantities of skilled labor and scientists:

$$l^*_{H,t} = 1 - a_{min} - \aleph \left((1-\eta)\, W^A_{H,L}\left(l^*_{H,t}\right)\right)^{-\frac{1+\beta}{\theta}}, \tag{56}$$

$$l^*_{H,A,t} = l^A_{H,A,t}(l_{H,t}, \pi_{t+1}) \Leftrightarrow W^*_{H,Y,A,t}\left(l_{H,t}, l^A_{H,A,t}(l_{H,t}, \pi_{t+1})\right) = 1. \tag{57}$$

From here we derive the new following proposition concerning the impacts of inflation on the Skill Premium and workers distribution.

**Proposition 3.** *In the presence of CIA constraints in the R&D sector, for a sufficiently high share of skilled workers an increase of expected inflation has a contemporaneous negative effect on the share of high-skilled workers and scientists and also the Skill Premium.*

*Proof.* Applying the implicit function theorem to (30), yields the following derivative, if $l_{H,t} > \tilde{l}_H$ :

$$\underbrace{\frac{\partial\, l^*_{H,t}}{\partial\, \pi_{t+1}}}_{-} = -\underbrace{\frac{\partial\, \underline{a}}{\partial\, W_{H,L}}}_{-}\underbrace{\frac{d\, W^A_{H,L,t}}{d\pi_{t+1}}}_{-}\left(\underbrace{\frac{\partial\, \underline{a}}{\partial\, W_{H,L}}}_{-}\underbrace{\frac{d\, W^A_{H,L,t}}{dl_{H,t}}}_{-} + 1\right)^{-1}. \tag{58}$$

In turn, considering that $l^*_H = l^*_H(\pi_{t+1})$ in (35) and the skill premium in (31) leads to the following derivatives, with the signals indicated below , if $l_{H,t} > \tilde{l}_H$:

$$\underbrace{\frac{\partial\, l^*_{H,A,t}}{\partial\, \pi_{t+1}}}_{-} = \underbrace{\frac{\partial l^A_{H,A,t}}{\partial l_{H,t}}}_{+}\underbrace{\frac{\partial\, l^*_{H,t}}{\partial\, \pi_{t+1}}}_{-} + \underbrace{\frac{\partial l^A_{H,A,L}}{\partial \pi_{t+1}}}_{-},$$

$$\underbrace{\frac{\partial\, W^*_{H,L,t}}{\partial\, \pi_{t+1}}}_{-} = \underbrace{\frac{\partial W^A_{H,L,t}}{\partial l_{H,t}}}_{-}\underbrace{\frac{\partial\, l^*_{H,t}}{\partial\, \pi_{t+1}}}_{-} + \underbrace{\frac{\partial W^A_{H,L,t}}{\partial \pi_{t+1}}}_{-} = \underbrace{\frac{d\, W^A_{H,L,t}}{d\pi_{t+1}}}_{-}\left(\underbrace{\frac{\partial\, \underline{a}}{\partial\, W_{H,L}}}_{-}\underbrace{\frac{d\, W^A_{H,L,t}}{dl_{H,t}}}_{-} + 1\right)^{-1}.$$

□

Finally, the labor share is still equal to (38) and we derive the following proposition concerning how it is affected by inflation.

**Proposition 4.** *In the presence of CIA constraints in the R&D sector, under certain conditions, for a sufficiently high share of skilled workers in the short and medium-run an increase of expected inflation increases the manufacturing labor share with ambiguous effects otherwise. In the long-run it does not has any effect.*

*Proof.* If we differentiate (38) with respect to inflation, we the same expression as in (39), with the only difference being on some signals, shown below:

$$\underbrace{\frac{\partial\, lsm_t^*}{\partial\, \pi_{t+1}}}_{+/-} = \underbrace{\frac{\partial\, lsm_{L,t}^*}{\partial\, l_{H,t}^*}}_{-} \underbrace{\frac{\partial\, l_{H,t}^*}{\partial\, \pi_{t+1}}}_{-} + \underbrace{\frac{\partial\, lsm_{L,t}^*}{\partial\, l_{H,A,t}^*}}_{+/-} \underbrace{\frac{\partial\, l_{H,A,t}^*}{\partial\, \pi_{t+1}}}_{-} \begin{cases} >0 & \text{if } \left\{ \begin{array}{c} l_{H,t} > \tilde{l}_H \wedge \\ \wedge\, (\lambda l_{H,Y,t} - \alpha l_{H,A,t}) > \\ > \frac{\mathcal{C}_3 \alpha l_{H,A,t}}{A_{t-1}^{\varphi-1} L_t^{\lambda} \mathcal{C}_2 l_{H,A,t}^{\lambda} l_{H,t}^{\lambda}} \end{array} \right\} \\ \gtreqless 0 & \text{otherwise} \end{cases} . \tag{59}$$

where are defined $\underbrace{\frac{\partial\, lsm_{L,t}^*}{\partial\, l_{H,t}^*}}_{+}$ and $\underbrace{\frac{\partial\, lsm_{L,t}^*}{\partial\, l_{H,A,t}^*}}_{+/-}$ as in proposition 2. Therefore, we have that

$$\frac{\partial lsm_{L,t}^*}{\partial \pi_{t+1}} \begin{cases} >0 & \text{if } l_{H,t} > \tilde{l}_H \ \wedge \ A_{t-1}^{\varphi-1} L_t^{\lambda} \frac{\mathcal{C}_2}{\mathcal{C}_3 \alpha} l_{H,A,t}^{\lambda-1} l_{H,t}^{\lambda} \left(\lambda l_{H,Y,t} - \alpha l_{H,A,t}\right) > 1 \\ \gtreqless 0 & \text{otherwise} \end{cases} .$$

In the long-run, $A_{t-1}$ tends to infinity and, therefore, $\lim_{A_{t-1}\to\infty} \frac{\partial\, lsm_t^*}{\partial\, \pi_{t+1}} = 0$. □

The intuition for this results follows immediately from the intuition provided for the analogous proposition.

## 3.3 Synthesis and comparison

We summarize the impacts of an increase of inflation through both alternative CIA constraints in Table 3, under the necessary analytical conditions. The main difference concerning its impact is driven by the manner through which the increase of inflation affects the labor market through each particular sector of the economy affected by liquidity constraints. On the one hand, through cash constrained consumers, it acts as positive supply shock for skilled workers due to its negative impacts being relatively more felt by low-skilled workers. On the other hand, through constrained R&D firms it acts as a negative demand shock for scientists and, therefore, skilled workers by increasing the relative costs of pursuing innovative activities. These effects lead to opposite reallocation of labor in the economy - an increase of both portions of workers in the case of the first and a decrease in the case of the second - which, in turn, have opposite effects on the labor share - it decreases it in the case of the first and increases it in the case of the second. However, in both cases the impact on the relative price is always negative, and this is reflected by a fall in the skill premium in both cases.

Table 3: Summary of the impacts of inflation on the labor distribution, skill premium and labor share.

| Effect | CIA constrained agents | Channels | | | | $W^*_{H,t}$ | $l^*_{H,A,t}$ | $l^*_{H,t}$ | $lsm^*_t$ |
|---|---|---|---|---|---|---|---|---|---|
| $\uparrow \pi_t$ | Consumers | | | $\Rightarrow\uparrow l^S_{H,t}$ | $\underset{(a)}{\Rightarrow}\downarrow W_{H,Y,L,t}$<br>$\underset{(a)}{\Rightarrow}\downarrow W_{H,A,L,t}$<br>$\Rightarrow\downarrow W_{H,Y,A,t}$ | $\underset{(a)}{\downarrow}$ | $\underset{(a)}{\uparrow}$ | $\underset{(a)}{\uparrow}$ | $\underset{(a)+(b)}{\downarrow}$ |
| | R&D firms | $\nRightarrow W_{H,Y,L,t}$<br>$\Rightarrow\downarrow W_{H,A,L,t}$<br>$\Rightarrow\uparrow W_{H,Y,A,t}$ | $\underset{(a)}{\Rightarrow}\downarrow W^A_{H,t}$<br>$\Rightarrow\downarrow l^A_{H,A,t}$ | $\Rightarrow\downarrow l^S_{H,t}$ | $\underset{(a)}{\Rightarrow}\uparrow W_{H,Y,L,t}$<br>$\underset{(a)}{\Rightarrow}\uparrow W_{H,A,L,t}$<br>$\Rightarrow\uparrow W_{H,Y,A,t}$ | | $\underset{(a)}{\downarrow}$ | $\underset{(a)}{\downarrow}$ | $\underset{(a)+(b)}{\uparrow}$ |

Note: $(a)$ if $l_{H,t} > \tilde{l}_H$, $(b)$ if $A^{\varphi-1}_{t-1} L^{\lambda}_t \frac{\mathcal{C}_2}{\mathcal{C}_3 \alpha} l^{\lambda-1}_{H,A,t} l^{\lambda}_{H,t} \left(\lambda l_{H,Y,t} - \alpha l_{H,A,t}\right) > 1$.

# 4 Conclusions

We present new evidence on the relationship between inflation and the labor share. Our results show that when significant effect is detected it is negative and only in the short-run. In the long-run, inflation tend to be non-significantly related to the labor share, whatever the set of controls considered.

We also investigate the theoretical effects of inflation through an overlapping generations endogenous growth model with cash-in-advance (CIA) constraints affecting either consumers or R&D firms. Inflation increases the supply of skilled workers through cash-in-advanced consumers by creating more incentives for low-skilled workers to pursue education and avoid its negative consequences, which are felt more strongly by this type of worker. This, in turn, under certain conditions, leads to a higher portion of scientists and a lower skill premium and labor share. Through R&D firms facing liquidity constraints, it decreases the demand for scientists and skilled workers, leading to opposite effects of each considered channels, except in what concerns the skill premium, which also drops. These results are consistent with the empirical evidence showing only short-run (negative and low) influence of inflation on the labor share. Skill-premium wise our results also shed light on the somewhat contradictory effects referenced in the literature. In our case the effect of inflation on the skill-premium is always negative only in the transitional dynamics (short-run).

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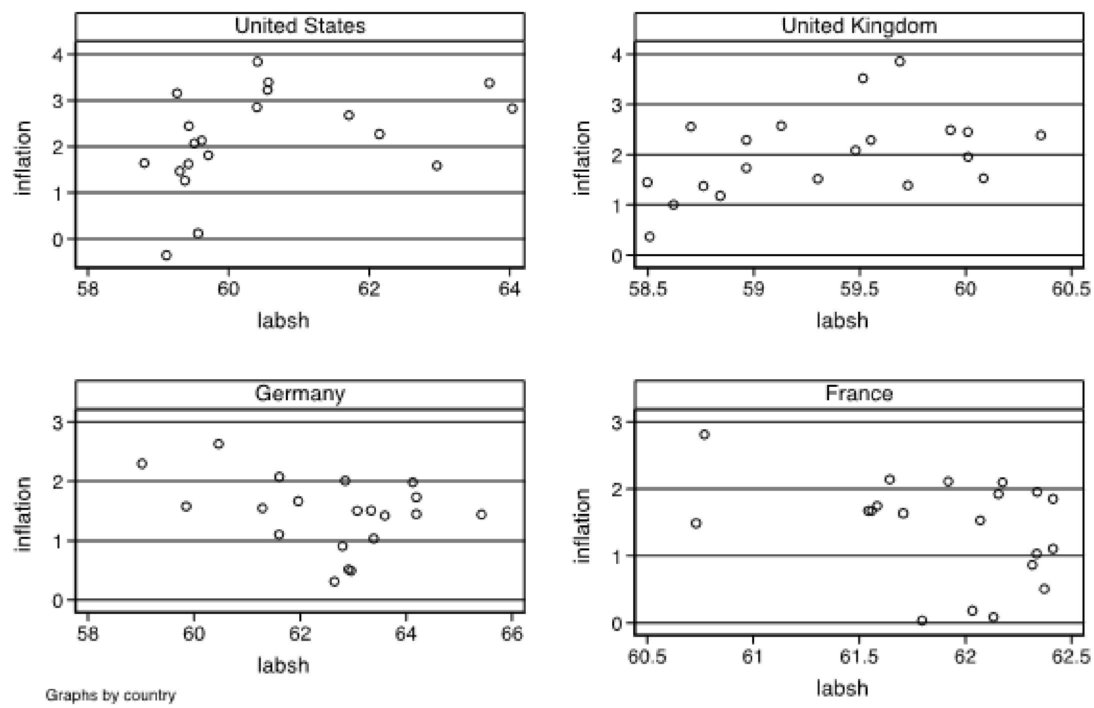
# A Appendix

## A.1 Complementary data and empirical tests

Although for the full dataset we can observe the negative correlation between our two variables of interest, there's still added interest in observing the behavior of individual countries as inflation can present different patterns in each. Different historical and cultural backgrounds as well as different policy stances from the central banking authority can explain some of these different patterns. To illustrate this, we show in Figure 2 the scatterplots between the labor share and inflation for four different countries, the United States, the United Kingdom, Germany, and France.

Figure 2: Scatterplot of all the datapoints by country




Before we proceed into the econometric estimations we check for the existence of unit roots in our data series. To this end we use the CIPS panel unit root test, following (Pesaran, 2007), which considers as the null hypothesis the existence of a unit root. The results of this test for the data series both in levels and in first differences can be found below in Table 2.

Table 4: Panel Unit Root Tests

| | No trend | | | | With Trend | | | |
|---|---|---|---|---|---|---|---|---|
| lags | 0 | 1 | 2 | 3 | 0 | 1 | 2 | 3 |
| labor share | 0.927 | 0.927 | 0.929 | 0.840 | 0.999 | 1.000 | 1.000 | 1.000 |
| $\log(h)$ | 0.966 | 0.283 | 0.512 | 0.063 | 1.000 | 0.750 | 0.988 | 0.990 |
| $k$ | 0.420 | 0.388 | 0.898 | 1.000 | 0.913 | 0.600 | 0.997 | 1.000 |
| $inflation$ | 0.000 | 0.000 | 0.000 | 0.001 | 0.000 | 0.000 | 0.015 | 0.389 |
| $unemployment$ | 0.968 | 0.016 | 0.576 | 0.899 | 0.981 | 0.337 | 0.408 | 0.981 |
| $\Delta$labor $share$ | 0.000 | 0.001 | 0.624 | 0.989 | 0.000 | 0.361 | 0.997 | 1.000 |
| $\Delta\log(h)$ | 1.000 | 0.977 | 0.995 | 1.000 | 0.917 | 0.183 | 0.905 | 0.996 |
| $\Delta k$ | 0.000 | 0.000 | 0.011 | 0.908 | 0.000 | 0.000 | 0.164 | 0.943 |
| $\Delta inflation$ | 0.000 | 0.000 | 0.000 | 0.010 | 0.000 | 0.000 | 0.008 | 0.840 |
| $\Delta unemployment$ | 0.000 | 0.001 | 0.001 | 0.156 | 0.000 | 0.515 | 0.212 | 0.859 |

Looking at the results presented in Table 2 we can see that the only data series that shows stationarity in levels is inflation, while the remaining variables reject the null of the existence of a unit root when the test is done for the variable in first differences, meaning they are I(1), the exception being the human capital variable which we find to be I(2). In the next step we test for the existence of a cointegration relationship between the labor share, inflation and unemployment. We test using the (Westerlund, 2005) test for which the null hypothesis is the absence of cointegration.

Table 5: Panel Cointegration Tests

| | No trend | With Trend |
|---|---|---|
| All panels | 0.2377 | 0.0166 |
| Some Penals | 0.0270 | 0.0001 |

As we can see from Table 3 there is evidence of the existence of a cointegration relationship involving the labor share, inflation and unemployment.

## A.2 Table with summarized effects of Lemma 2

Table 6: Summary of the impacts of changes in the portion of skilled labor and scientists

| | | | | | |
|---|---|---|---|---|---|
| $\uparrow l_{H,A,t}$ | $\not\Rightarrow L_{L,t}$ | | $\not\Rightarrow w_{L,t}$ | $(ii-b), (ii-a) \rangle \Rightarrow\downarrow w_{L,t}$ | $\Rightarrow\uparrow W_{H,Y,L,t}$ |
| | $\Rightarrow\sim L_{H,t}$ | $\underset{(ii)}{\Rightarrow}\downarrow L_{H,Y,t}$ | $\underset{(ii-b)}{\Rightarrow}\downarrow w_{L,t}$ | $(i-b), (ii-a) \rangle \Rightarrow\uparrow w_{H,Y,t}$ | $\Rightarrow\downarrow W_{H,A,L,t}$ |
| | | | $\underset{(ii-a)}{\Rightarrow}\uparrow w_{H,Y,t}$ | $(ii-c), (i-a) \rangle \Rightarrow\downarrow w_{H,A,t}$ | $\Rightarrow\uparrow W_{H,Y,A,t}$ |
| | | | $\underset{(ii-c)}{\Rightarrow}\downarrow \Pi_{i,t} \Rightarrow\downarrow \dot{A}_t \Rightarrow\downarrow w_{H,A,t}$ | | |
| | | $\underset{(i)}{\Rightarrow}\uparrow L_{H,A,t}$ | $\underset{(i-b)}{\Rightarrow}\uparrow A_t \Rightarrow\uparrow w_{H,Y,t}$ | | |
| | | | $\underset{(i-a)}{\Rightarrow}\downarrow \dot{A}_t \Rightarrow\downarrow w_{H,A,t}$ | | |
| $\uparrow l_{H,t}$ | $\underset{\boldsymbol{(iv)}}{\Rightarrow}\downarrow \boldsymbol{L_{L,t}}$ | | $\underset{\boldsymbol{(iv-a)}}{\Rightarrow}\uparrow \boldsymbol{w_{L,t}}$ | $(iv-a), (iii-a) \rangle \Rightarrow\uparrow w_{L,t}$ | $\underset{(a)}{\Rightarrow}\downarrow W_{H,Y,L,t}$ |
| | $\Rightarrow\uparrow \boldsymbol{L_{H,t}}$ | $\underset{\boldsymbol{(iii)}}{\Rightarrow}\uparrow \boldsymbol{L_{H,Y,t}}$ | $\underset{\boldsymbol{(iii-a)}}{\Rightarrow}\uparrow \boldsymbol{w_{L,t}}$ | $(i-b), (iii-b) \rangle \Rightarrow\updownarrow w_{H,Y,t}$ | $\underset{(a)}{\Rightarrow}\downarrow W_{H,A,L,t}$ |
| | | | $\underset{\boldsymbol{(iii-b)}}{\Rightarrow}\downarrow \boldsymbol{w_{H,Y,t}}$ | $(iii-c), (i-a) \rangle \Rightarrow\uparrow w_{H,A,t}$ | $\Rightarrow\downarrow W_{H,Y,A,t}$ |
| | | | $\underset{\boldsymbol{(iii-c)}}{\Rightarrow}\uparrow \boldsymbol{\Pi_{i,t}} \Rightarrow\uparrow \boldsymbol{\dot{A}_t} \Rightarrow\uparrow \boldsymbol{w_{H,A,t}}$ | | |
| | | $\underset{(i)}{\Rightarrow}\uparrow L_{H,A,t}$ | $\underset{(i-b)}{\Rightarrow}\uparrow A_t \Rightarrow\uparrow w_{H,Y,t}$ | | |
| | | | $\underset{(i-a)}{\Rightarrow}\downarrow \dot{A}_t \Rightarrow\downarrow w_{H,A,t}$ | | |

Note: $(a)$ if $l_{H,t} > \tilde{l}_H$