Wage Inequality and Lobbying: a directed technical change approach

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Abstract

We devise a generalized Directed Technical Change growth model in which firms spend resources in lobbying activity. As expected, the presence of lobbying distorts the skill premium and economic growth. Lobbying also contributes to a lower technological-knowledge bias toward the skill-sector and constitutes a possible explanation for the diverging empirical evidence on the relationship between the skill premium and the relative supply of skills. An increase in the relative lobbying power of the skilled intensive intermediate goods firms can lead to an increase or decrease in the skill premium, depending on the elasticity of substitution between the skilled and unskilled sectors. Lobbying also introduces possibility of a dual economy, with two different steady states, one characterized by low growth and another by high growth, depending on a threshold level of the lobbying power and on the elasticity of substitution. Quantitative exercises show that lobbying can indeed be quite important in distorting the skill premium and the economic growth.

Keywords: Directed technical change; lobbying power; inefficiency; economic growth; wage inequality; quantitative implications.

JEL Classification: J31, P16, O30, O41.

"*Lobbying organizations(...) spent $1.64 billion – a figure which increased steadily until 2010*." Washington Post, 21th April 2015.

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1 Introduction

In an article published in April 2015 in the Washington Post, Phillip Bump stressed that since 2001, the USA firms have spent more in lobbying activity than the value paid in salaries for members of the Congress and their staff. In the economic literature, lobbying activity has been seen as a consequence and not as a cause of inequality (Esteban and Ray, 2006). However, lobbying has been pointed out to negatively affect technologies adoption (Comin and Hobjin, 2009). With firms allocating funds to lobbying, their relative profitability may be affected and, as a consequence, so, too, the R&D decisions that drive the technological-knowledge change and bias.[1] In an environment in which wage inequality in favour of skilled labour (skill premium) is determined not only by the relative supply of skills but also by the technological-knowledge bias, as in the Directed Technical Change (DTC) literature (Acemoglu, 2002), lobbying is a natural candidate to affect the skill premium through the technological-knowledge bias. In this paper, we pursue this line of reasoning, which also fills a gap in the literature, which has not considered lobbying in an endogenous DTC model.

We thus introduce lobbying in a generalized DTC model, considering the division of the economy into unskilled and skilled sectors. The output of each sector is produced with specific labour, unskilled or skilled, and a continuum set of specific available complementary non-durable differentiated intermediate goods. As a result, we introduce an alternative explanation to the diverging patterns of the skill premium and skills supplies around the world. In fact, Autor (2014) recognized that besides market effects, inequality may depend on changing social norms, growing corporate misgovernance, slackening regulatory oversight, or increasing political capture of the policy making process by elites (e.g., lobbying).

The generalized DTC literature (e.g., Bound and Johnson, 1992; Katz and Murphy, 1992; Juhn et al., 1993) works out the contradiction between the rise in the skill premium and the increase in the relative supply of skills. The argument is that technological-knowledge change induces an increase in the relative demand of more-skilled workers that exceeds the increase in the relative supply, thereby increasing the skill premium. Acemoglu (1998, 2002) and Acemoglu and

[1] The technological-knowledge change represents the overall process of invention / innovation as a result of R&D activity, and the technological-knowledge change can be biased/directed for some particular sector.

Zilibotti (2001) further enhance this literature by considering that technological-knowledge change responds to shifts in labour endowments. When the supply of a type of labour increases, the market for technologies that complement it broadens (market-size channel), and this creates additional incentives for R&D aimed at those technologies. Consequently, technological-knowledge changes toward those technologies, which in turn increases the demand for the complementary type of labour. Hence, the rise in the skill premium would be a direct consequence of the increase in the relative supply of more-skilled workers.

However, some empirical evidence seems to contradict the explanation proposed by the generalized DTC literature. Acemoglu (2003), for example, documents a decline in the skill premium in The Netherlands between the early 1980s and the mid-1990s, in a scenario with relative increase of skills, and an increase in the skill premium in Canada between the late 1980s and the late 1990s, in a scenario with stable relative supply of skills. Moreover, data from developing countries reveals additional problematic evidence. Crinó (2005), for example, shows that Hungary and the Czech Republic experienced an increase in the skill premium between 1993 and 2004, while at the same time the relative employment of more skilled workers declined. Robertson (2004), among others, detects that wage differential between the 90th and 10th wage percentiles decreased in Mexico between 1994 and 2002, even with the relative increase of highly-educated workers. And Zhu and Trefler (2005), for example, reveal that the same situation occurred in Bolivia, South Korea, and the Philippines. We conjecture that a lobbying mechanism working within the generalized DTC framework can contribute to conciliate this diverse evidence with theory.

Indeed, firms' lobbying establishes barriers to the competitive allocation of resources or alternatively prevents the establishment of such barriers. The misallocation of resources can be induced by: barriers to competition both international (e.g., export subsidies, import tariffs, and quotas) and domestic (e.g., entry barriers, inefficient financial system, and large subsidized state enterprises), as in Cole et al. (2005); social infrastructure, i.e., institutions and government policies, as in Hall and Jones (1999); technology-adoption costs, as in Parente and Prescott (1994, 1999); barriers that alter the entry and exit decisions of firms, as in Bergoeing et al. (2010); or misallocation in the intermediate goods at the firm level, as in Jones (2011). This paper also fits in with the literature that emphasizes the role of institutional arrangements such as the extent of special interest lobbying on the economic performance of a country (e.g., Olson, 1982, 1996). This literature argues that economic policies and institutions determine the extent to which nations attain their potential; i.e., poor policies and institutions create a set of incentives such that nations are not operating on their production frontiers.

Our endogenous R&D growth model is closely related to the contributions of Acemoglu (1998, 2002) and Acemoglu and Zilibotti (2001), which stress the market-size channel, as well as the contributions of Afonso (2006, 2008), which, in turn, highlight the price channel due to the removal of scale effects. The former channel encourages innovation toward the sector, skilled or unskilled, with larger market and the latter channel directs innovation toward the sector with higher price. We are able to accommodate both channels, by considering the possibility of different degrees of scale effects, and in addition we take into account the misallocation of resources that also affect the direction of technological-knowledge change. This is how we intend to accommodate the different paths of both the skill premium and the relative supply of workers that have been documented empirically.

Some general results should be emphasized. The technological-knowledge bias depends: positively on both the relative importance of the skilled-sector in the production of the aggregate final good and the relative productivity of developing skilled sector complementary intermediate goods; positively, null, or negatively on the relative supply of skills. Moreover, the relative profitability of the intermediate-goods firms in the H-sector depends on the relative lobbying strength of these firms: if it is low then the intermediate-goods firms in the unskilled L-sector have the advantage in lobbying; if it is big enough then the intermediate-goods firms in the H-sector have the advantage in lobbying. Regardless of the case, when the relative lobbying strength of the intermediate-goods firms in the H-sector increases, then the relative profitability of the intermediate-goods firms in the H-sector also increases. Moreover, due to lobbying, the technological-knowledge bias becomes less directed toward the skilled sector, thus producing a wrong mix of goods in the long run. In turn, the skill premium is now positively related with the lobbying strength of the intermediate-goods firms in the H-sector, but just under substitutability between sectors and positively, null, or negatively related with the relative supply of skills.

We also take the model to data, through a calibration exercise that highlights our main theoretical results, showing their empirical plausibility. This is also one of the few examples of a Directed Technical Change model that is calibrated and presents quantitative results. This may be attributed to the difficulty of finding data related to skilled and unskilled sectors, both in production of goods and innovations, which has prevented most of the previous contributions from presenting quantitative results.

The paper is organized as follows. Section 2 presents the model. Section 3 presents a calibration exercise that shows the quantitative importance of considering lobbying in the DTC framework to study its influence on wage inequality and growth. Section 4 concludes.

2 Theoretical model

This section describes the economic model, emphasising the interactions among economic agents, and the dynamic general equilibrium in which (i) households and firms are rational and solve their problems, (ii) free-entry R&D conditions are met, and (iii) markets clear. We start by considering the optimizing behaviour of the infinitely-lived households that inelastically supply labour, unskilled (L) or skilled (H), maximize utility of consumption, and invest in the firm's equity. Then, we describe the productive side, stressing the maximization problem faced by final-good firms, intermediate-good firms, and R&D firms.

The inputs of the aggregate or composite final good (or numeraire) are two intermediate final goods, each supplied by a large number of competitive firms: one is produced in the unskilled sector (L-sector) and the other is produced in the skilled sector (H-sector), and each uses specific labour, L or H, and a continuum of specific non-durable intermediate goods. Each intermediate-goods sector consists of a continuum of industries, $j \in [0, N_j(t)]$, $j = L, H$, and there is monopolistic competition: the monopolist in industry j uses a design sold by the R&D sector (domestically protected by a perpetual patent), and aggregate final good to produce a non-durable intermediate good at a price chosen to maximize profits. That is, imperfectly competitive firms buy designs (technological knowledge) in the R&D sector to produce intermediate goods, which can complement the inputs used by perfectly competitive final-goods firms in either the L-sector or the H-sector. Therefore, the relative productivity of the technological knowledge depends on the sector in which it is employed. In the R&D sector there is free entry and each potential entrant devotes aggregate final good to produce/invent successful horizontal designs, which are then supplied to a monopolist firm in a new intermediate-goods industry; i.e., the R&D sector allows increasing the number of intermediate-goods industries $N(t)$ and thus the technological knowledge. Lobbying activity is incorporated in the model considering that the price paid by consumers of intermediate goods (the producers of final goods) does not correspond to the price received by the respective producers, and by considering that the lobbying power of intermediate-goods firms differs between sectors. Infinitely-lived households inelastically supply labour, skilled and unskilled, maximize utility obtained with the consumption of the homogenous final good, and earn income from labour and from investments in financial assets.

2.1 Technology and preferences

The economy is populated by a fixed number of infinitely-lived households who consume and collect income from investments in financial assets (equity) and from labour. Households inelastically supply labour to two intermediate final-goods sectors: the unskilled (L-sector), L, and the skilled (H-sector), H. Thus, total labour supply, unskilled and skilled, is exogenous and constant. We regard the exogenous and constant labour supply as mainly instrumental to the isolation of the other effects of the lobbying activity, namely in the change and bias of the technological knowledge and, consequently, in the skill premium and in the economic growth.

We assume that consumers have perfect foresight concerning the technological-knowledge change over time, $\frac{\dot{N}_j(t)}{N_j(t)}$, $j = L, H$, and choose the path of final-good aggregate consumption $[C(t)]_{t\geq 0}$ to maximise discounted lifetime utility $U = \int_0^\infty \left(\frac{C(t)^{1-\theta}-1}{1-\theta}\right) e^{-\rho t} dt$, where $\rho > 0$ is the subjective discount rate, ensuring that $U(.)$ is bounded away from infinity if C were constant over time, and $\theta > 0$ is the inverse of the intertemporal elasticity of substitution, subject to the flow budget constraint $\dot{a}(t) = r(t) \cdot a(t) + w_L(t) \cdot L + w_H(t) \cdot H - C(t) + \tau - b$, where a denotes households' real financial assets holdings, w_j is the wages for labour employed in the final j-sector, and τ and b (with $\tau = b$) denote, respectively, the amount of resources obtained and paid with lobbying activity. The initial level of wealth $a(0)$ is given and the non-Ponzi games condition $lim_{t\to\infty} e^{-\int_0^t r(s)ds} a(t) \geq 0$ is imposed. The optimal consumption path Euler equation,

$$\frac{\dot{C}(t)}{C(t)} = \frac{1}{\theta} \cdot (r(t) - \rho), \tag{1}$$

and the transversality condition, $\lim_{t\to\infty} e^{-\rho t} \cdot C(t)^{-\theta} \cdot a(t) = 0$, are standard. The aggregate financial wealth held by households is composed by equity of intermediate goods producers $a(t) = a_L(t) + a_H(t)$, where $a_j(t) = N_j(t)V_j(t)$, $j = L, H$, where, remember, N_j is the number of available types of intermediate goods and thus the technological-knowledge frontier in each j-sector, and V_j is the present value of monopoly profits seized by each intermediate good producer – see the analysis below. Taking time derivatives and comparing with the flow budget constraint above, the aggregate flow budget constraint is equivalent to the final product market equilibrium condition

$$Y(t) = C(t) + X(t) + Z(t), \tag{2}$$

where $Y(t)$ is the aggregate final good (or numeraire), $X(t)$ is the total investment in production of intermediate goods, and $Z(t)$ are the aggregate R&D expenditures. Final-good producers are competitive and Y is produced with a CES aggregate production function of unskilled and skilled final goods:

$$Y(t) \quad = \left[\chi_L Y_L(t)^{\frac{\varepsilon-1}{\varepsilon}} + \chi_H Y_H(t)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \quad , \quad \varepsilon \in (0, +\infty) \, , \tag{3}$$

where: Y_L and Y_H are the total outputs of the L-sector and the H-sector, respectively (i.e., the intermediate final goods); χ_L and χ_H, with $\chi_L + \chi_H = 1$, are the distribution parameters, measuring the relative importance of the inputs; $\varepsilon \geq 0$ is the elasticity of substitution between the two inputs in the production of the aggregate final good, wherein $\varepsilon > 1$ ($\varepsilon < 1$) means that the inputs from the sectors are gross substitutes (complements) in the production of Y.[2] Without loss of generality, we normalize the price of the aggregate final good at unit,[3]

$$P_Y \equiv 1 = \left[\chi_L^{\varepsilon} P_L^{1-\varepsilon} + \chi_H^{\varepsilon} P_H^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \quad , \tag{4}$$

where the right hand side of this expression, $\left[\chi_L^{\varepsilon} P_L^{1-\varepsilon} + \chi_H^{\varepsilon} P_H^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$, is the unit cost of production since P_L and P_H are the prices of the outputs of, respectively, the L-sector and the H-sector. This normalization and the assumption of competitive final-good firms imply the following maximization problem: $Max\, \Pi = Y - P_L Y_L - P_H Y_H$. From the first-order conditions, we obtain the following expression for the relative price of the H-sector in terms of the L-sector:

[2]When $\varepsilon = 0$, there is no substitution between Y_L and Y_H, and the production function is Leontieff. When $\varepsilon = 1$, the production function is Cobb-Douglas. When $\varepsilon = +\infty$, Y_L and Y_H are perfect substitutes, and the production function is linear.

[3]To simplify notations, we suppress the time argument t and will do so throughout as long as this causes no confusion.

$$\frac{P_H}{P_L} = \frac{\chi_H}{\chi_L}\left(\frac{Y_H}{Y_L}\right)^{-\frac{1}{\varepsilon}}, \tag{5}$$

which is the usual relative inverse demand curve that, as expected, has a negative slope. Hence, the relative price of the H-sector is a decreasing function of the relative output of the sector, $\frac{Y_H}{Y_L}$. Moreover, the relative importance of the sector's output, $\frac{\chi_H}{\chi_L}$, which serves as an input in final-good production, makes the relative price higher.

Concerning the output of each intermediate final-goods sector, we consider that the output of the j-sector, $j = L, H$, is produced with specific labour, $W_L = L$ and $W_H = H$, and a continuum set of available complementary non-durable differentiated intermediate goods x_j in the $(0, N_j]$. In order to solve the model analytically, we use the Dixit-Stiglitz constant elasticity structure for production in the intermediate final-goods sector:

$$Y_j = A \cdot W_j^{\alpha} \cdot \int_0^{N_j} x_j(n)^{1-\alpha} dn, \; j = L, H, \tag{6}$$

where: A is a positive exogenous variable representing the level of productivity, dependent on the country's institutions; $1-\alpha$ and $\alpha \in [0,1]$ are the intermediate-goods and the labour shares, respectively; N_L and N_H represent the number of already available intermediate goods, which measure the technological knowledge and can be interpreted as the extent of specialization (e.g., Gancia and Bonfiglioli, 2008); i.e., the former (latter) increases the productivity of L (H) and hence the output of the L-sector (H-sector). The maximization problem of the firms in the j-sector is $Max\, P_j Y_j - w_j W_j - \int_0^{N_j} P_{x_j}(n) x_j(n) dn, \; j = L, H,$ where P_{x_j} is the price of the j-sector (labour complementary) intermediate good and w_j is the wage paid for j.[4] From the first-order conditions we obtain:

$$w_j = P_j \cdot \alpha \cdot A \cdot W_j^{\alpha-1} \cdot \int_0^{N_j} x_j(n)^{1-\alpha} dn = \alpha \frac{P_j Y_j}{W_j}, \; j = L, H, \; W_L = L, \; W_H = H, \tag{7}$$

$$P_{x_j}(n) = P_j \cdot (1-\alpha) \cdot A \cdot W_j^{\alpha} \cdot x_j(n)^{-\alpha} \Rightarrow x_j(n) = \left(\frac{P_j \cdot (1-\alpha) \cdot A}{P_{x_j}(n)}\right)^{\frac{1}{\alpha}} W_j, \; j = L, H. \tag{8}$$

[4] Since the (labour complementary) intermediate goods depreciate fully after use, the optimizations for the j-sector, $j = L, H$, are static.

From (7), the wage paid for the labour employed in a particular sector is equal to the value of the respective marginal contribution to the production in the sector.[5] In turn, in (8) we have the demand for the n-type intermediate good used in each sector, L and H, which depends on three factors: (i) the price of the respective output, P_L or P_H, since, all things being equal, the higher the price of the output the higher the demand for n; (ii) the price of the intermediate good, $P_{x_L}(n)$ or $P_{x_H}(n)$, since, all things being equal, the demand for n is a decreasing function of its own price; and (iii) the respective employed labour level, L or H, since, all things being equal, the higher the labour level, the higher the demand for n, given that larger supply of labour raises the productivity of n, thereby increasing its demand.

We now need to look at the profit maximization problem of the intermediate-good firms. Once the intermediate-good firm has a new design, sold by the R&D sector, it can retain a perpetual monopoly over the use of this design. Thus, the flow of the monopolist's operational profit, which sells its good to the j-sector, at a point of time is $\pi_j = \left[P_{x_j}(n) - \eta\right] \cdot x_j(n)$, $j = L, H$, and the present value of the returns from the operation is $V_j = \int_t^{\infty} \pi_j(v) \cdot e^{-\int_t^v r(\omega)d\omega} dv$, $j = L, H$, where r is the interest rate. Hence, the monopolist faces the demand curve (8), and solves the following problem:

$$max \int_t^{\infty} \left[P_{x_j}(v) - \eta\right] \cdot x_j(v) \cdot e^{-\int_t^v r(\omega)d\omega} dv, \ j = L, H, \tag{9}$$

and reaches:

$$P_{x_j}(n) = P_x = \frac{\eta}{1-\alpha} = 1, \tag{10}$$

by considering, as Acemoglu (2002), that $\eta = 1-\alpha$, which simplifies the notation without any loss of generality. Hence, the profit maximizing price of intermediate goods in the benchmark case is equal to one unit of the aggregate final good, implying that one unit of intermediate good employed by either sector is exchanged one for one with the aggregate final good. Indeed, the isoelastic nature of the demand for n implies that each monopolist sets a constant markup over the marginal cost: $1 > \eta$ since $\alpha \in [0, 1]$. Thus, in each sector, each monopolist charges the same price, produces the same amount, and has the same profit at every period, and thus the present value of the monopoly operational profit is the same for each firm.

[5] The result about wage setting follows from basic microeconomics principles on the assumption that the labour market is competitive.

Before introducing the R&D sector to consider endogenous technological knowledge, we analyse the productive equilibrium under constant technological knowledge. We begin by substituting the equilibrium price of intermediate goods in (10) into the intermediate-goods demand functions in (8), yielding:

$$x_j(n) = [P_j \cdot (1-\alpha) \cdot A]^{\frac{1}{\alpha}} \cdot W_j,\ j = L, H,\ W_L = L,\ W_H = H, \tag{11}$$

As expected, the equilibrium intermediate-goods demand functions in (11) imply that the demanded quantities in equilibrium do not depend on the identity of the intermediate good. What matters is the sector's output price and labour level in which the intermediate good is used. However, we intend to analyse what happens if resources directed to intermediate-goods production are the outcome of a contest between firms producing for both sectors, H-sector and L-sector (interplay between firms).[6] One can envision this contest as an activity whereby firms lobby individuals, whose activity allows them to establish barriers to the correct allocation of resources or, alternatively, to prevent the establishment of barriers (e.g., Grossmann and Steger, 2008; Mathur et al., 2013; Cothren and Radhakrishnan, 2017). As a result, intermediate-goods producers can either move away from or to defend an allocation of resources that tends to the benchmark case without lobbying activity. To accommodate the lobbying activity hypothesis, we consider that, due to the lobbying costs, the price of intermediate goods paid by the producers of final goods in (10), which determines the demand in (11), may not correspond to the price received by the respective producers. We assume that the price received by producers of intermediate goods used in: (i) the H-sector can vary between 1 in (10) and β, which is the price that can be received after bearing lobbying costs; (ii) the L-sector can vary between 1 in (10) and γ, which is the price that can be received after bearing lobbying costs. From this, substituting (11) into the flow of the monopolist's operational profit implies:

$$\pi_j = \alpha \cdot P_j^{\frac{1}{\alpha}} \cdot (1-\alpha) \cdot A^{\frac{1}{\alpha}} \cdot W_j,\ j = L, H,\ W_L = L,\ W_H = H, \tag{12}$$

or, due to lobbying cost, we assume that the price of intermediate goods paid by the producers of final goods and the price received by the respective producers diverge from each other, which is captured by $\beta > 0$ and $\gamma > 0$, considering that under lobbying $P_{x_H} = 1 + \alpha(\beta - 1)$ and $P_{x_L} = 1 + \alpha(\gamma - 1)$:

$$\pi_H^{\mathcal{L}} = \beta \cdot \alpha \cdot P_H^{\frac{1}{\alpha}} \cdot (1-\alpha) \cdot A^{\frac{1}{\alpha}} \cdot H,\ or\ \pi_L^{\mathcal{L}} = \gamma \cdot \alpha \cdot P_L^{\frac{1}{\alpha}} \cdot (1-\alpha) \cdot A^{\frac{1}{\alpha}} \cdot L, \tag{13}$$

and from now on the superscript $\mathcal{L}$ designates "with lobbying activity".

[6]In a context in which labour supply is exogenous and constant, the sectors, H and L, can gain preponderance if the technological-knowledge progress is directed in favour of a sector and, as shown later on, that bias is conditioned by the lobbying activity.

Bearing also in mind (11), the equation (6) can be re-written as:

$$Y_j = P_j^{\frac{1-\alpha}{\alpha}} \cdot (1-\alpha)^{\frac{1-\alpha}{\alpha}} \cdot A^{\frac{1}{\alpha}} \cdot W_j \cdot N_j, \; j = L, H, \; W_L = L, \; W_H = H, \qquad (14)$$

which indicates that the equilibrium quantity produced in each intermediate final-goods sector depends positively on the sector's (i) output price, P_L or P_H, (ii) labour level, L or H, (iii) technological-knowledge level, N_L or N_H, as well as on (iv) the exogenous productivity, A. Now, from (14) and (5), the relative price of the H-sector can be rewritten as:

$$\frac{P_H}{P_L} = \left[\left(\frac{\chi_H}{\chi_L} \right)^{\varepsilon\alpha} \left(\frac{H \cdot N_H}{L \cdot N_L} \right)^{-\alpha} \right]^{\frac{1}{1+\Omega}}, \qquad (15)$$

where $1 + \Omega$, with $\Omega \equiv (\varepsilon - 1)\,\alpha$, is the elasticity of substitution between the two intermediate final goods in the aggregate final good production; in fact, $1 + \Omega > 1$ only occurs when $\varepsilon > 1$. From (15), the relative price of the H-sector intermediate input depends: (i) positively on the relative importance of the H-sector intermediate input in the production of the aggregate final good, $\frac{\chi_H}{\chi_L}$; (ii) negatively on the relative supply of H, $\frac{H}{L}$, and on the technological-knowledge bias between sectors, $\frac{N_H}{N_L}$.

To reach the relative wage; i.e., the intra-country wage inequality measure, $\frac{w_H}{w_L}$, with constant technological knowledge, equations (14) and (15) should be substituted into (7), obtaining then the following expression:

$$\frac{w_H}{w_L} = \left[\left(\frac{\chi_H}{\chi_L} \right)^{\varepsilon} \left(\frac{H}{L} \right)^{-1} \left(\frac{N_H}{N_L} \right)^{\Omega} \right]^{\frac{1}{1+\Omega}}, \qquad (16)$$

and thus the intra-country wage inequality depends: (i) positively on $\frac{\chi_H}{\chi_L}$ and on $\frac{N_H}{N_L}$ if $\varepsilon > 1$; (ii) negatively on $\frac{H}{L}$ and on $\frac{N_H}{N_L}$ if $\varepsilon < 1$.

By combining (12), (13), and (15), the equilibrium expression for the relative profitability of the intermediate-goods firms in the H-sector is:

$$\frac{\pi_H}{\pi_L} = \left[\left(\frac{\chi_H}{\chi_L} \right)^{\varepsilon} \left(\frac{H}{L} \right)^{\Omega} \left(\frac{N_H}{N_L} \right)^{-1} \right]^{\frac{1}{1+\Omega}}, \qquad (17)$$

or

$$\frac{\pi_H^{\mathcal{L}}}{\pi_L^{\mathcal{L}}} = \frac{\beta}{\gamma} \cdot \frac{\pi_H}{\pi_L}, \qquad (18)$$

and thus, $\frac{\pi_H}{\pi_L}$ depends: (i) positively on $\frac{\chi_H}{\chi_L}$ and on $\frac{H}{L}$ if $\varepsilon > 1$; (ii) negatively on $\frac{N_H}{N_L}$ and on $\frac{H}{L}$ if $\varepsilon < 1$, while $\frac{\pi_H^{\mathcal{L}}}{\pi_L^{\mathcal{L}}}$ relies on the interplay between firms in both sectors, $\frac{\beta}{\gamma}$, and on $\frac{\pi_H}{\pi_L}$.

2.2 Directed technological change

We now investigate how the results of the model are affected by considering that the rate and the direction of the technological knowledge are both endogenous. In the R&D sector there is free entry and each potential entrant devotes aggregate final good to produce a successful design, which is protected by a system of patents and allows the introduction of a new intermediate good; i.e., a new firm in a new industry n. This new variety complements either H or L, but not both; i.e., we adopt a horizontal lab-equipment R&D specification (e.g., Acemoglu, 2002). Hence, innovations evolve over time according to the following equation of motion:

$$\dot{N}_j(t) = \sigma_j \cdot Z_j(t) \cdot W_j^{-\delta},\ j = H, L,\ W_L = L, W_H = H, \tag{19}$$

where σ_j is the productivity of the R&D activity in j-sector and $\frac{\sigma_H}{\sigma_L}$ can be interpreted as a measure of relative advantage to entry through horizontal innovation into the H-sector. Moreover, Z_j is the flow of aggregate final-good resources devoted to R&D directed at discovering a new intermediate good to be used in the j-sector. Thus, total R&D expenditure, Z, satisfies $Z = Z_H + Z_L$, and $Z_j(t) = \frac{\dot{N}_j(t)}{\sigma_j \cdot W_j^{-\delta}}$. Finally, given that scale effects are often considered implausible (e.g., Jones, 1995a, b), $W_j^{-\delta}$, $\delta \geq 0$, implies that an increase in market scale, measured by H or L, dilutes the effect of R&D outlays on the innovation rate, due to coordination, organisational and transportation costs related to market size (e.g., Afonso, 2012), which, as we can see below, can partially $(0 < \delta < 1)$, totally $(\delta = 1)$, or over counterbalance $(\delta > 1)$ the scale benefits on profits and thus allows us to remove scale effects on the economic growth rate. This contrasts with the usual knife-edge assumption that either $\delta = 0$ or $\delta = 1$ (e.g., Barro and Sala-i-Martin, 2004, ch. 6).

We assume that there is free entry into R&D, such that, in equilibrium, the cost of discovering a new variety, Z_j, is also its price, V_j, which corresponds to the present value of monopoly profits; i.e., $V_j(t) = \int_t^\infty \pi_j(v) \cdot e^{-\int_t^v r(\omega)d\omega}dv$ or $V_j^{\mathcal{L}}(t) = \int_t^\infty \pi_j^{\mathcal{L}}(v) \cdot e^{-\int_t^v r(\omega)d\omega}dv$, $j = H, L$, Differentiating both sides of the latter expression with respect to t yields:

$$V_j = \frac{\pi_j}{r} + \frac{\dot{V}_j}{r},\ or\ V_j^{\mathcal{L}} = \frac{\pi_j^{\mathcal{L}}}{r} + \frac{\dot{V}_j}{r}, j = H, L, \tag{20}$$

Along the balanced growth path (BGP), $V_j(t)$ is constant for all t, $\dot{V}_j(t) = 0$, and the interest rate is constant; that is, on the BGP, the interest rate is identical to the ratio of the profit flow to the lump-sum cost of discovery: $r(t) = \frac{\pi_j(t)}{V_j(t)}$ or $r(t) = \frac{\pi_j^{\mathcal{L}}(t)}{V_j(t)}$. Hence, bearing in mind (12),

$$V_j = \frac{\alpha \cdot P_j^{1/\alpha} \cdot (1-\alpha) \cdot A^{\frac{1}{\alpha}} \cdot W_j}{r(t)},\ or\ V_H^{\mathcal{L}} = \frac{\beta \cdot \alpha \cdot P_H^{\frac{1}{\alpha}} \cdot (1-\alpha) \cdot A^{\frac{1}{\alpha}} \cdot H}{r},\ or\ V_L^{\mathcal{L}} = \frac{\gamma \cdot \alpha \cdot P_L^{\frac{1}{\alpha}} \cdot (1-\alpha) \cdot A^{\frac{1}{\alpha}} \cdot L}{r}\ j = H, L, \tag{21}$$

i.e., the present value of monopoly profits depends: (i.a) positively on the product price of the sector in which the intermediate good is used, P_j, since it increases the value of the marginal product of all factors, including that of intermediate goods, thus encouraging firms to rent more intermediate goods and raising the instantaneous profits of the monopolist (price channel); (i.b) positively on production firms' employment, j, since it implies more labour to use intermediate goods, increasing demand, and thereby raises the profits (market-size channel); (ii) negatively on the rental price of capital, r, since it raises the discount rate for the future profit flow, and so reduces the present value. Strong lobbying activity, reflected in lower β and γ, also penalize the present value of monopoly profits. Hence, it should be stressed that, for example, the greater V_H is relative to V_L, the greater are the incentives to develop H-complementary intermediate goods, N_H, rather than L-complementary intermediate goods, N_L, and without the interplay between firms in both sectors there are only two forces determining the technological-knowledge bias, which are the price and the market-size channels since the incentives to invent technologies are greater when, respectively, goods are expensive and the market for the technology is larger:

$$\frac{V_H}{V_L} = \frac{\pi_H}{\pi_L} = \underbrace{\left(\frac{P_H}{P_L}\right)^{\frac{1}{\alpha}}}_{Price\ channel} \underbrace{\left(\frac{H}{L}\right)}_{Market-size\ channel}. \tag{22}$$

However, these two forces are enhanced by the interplay between firms in both sectors so that in this case:

$$\frac{V_H^{\mathcal{L}}}{V_L^{\mathcal{L}}} = \frac{\pi_H^{\mathcal{L}}}{\pi_L^{\mathcal{L}}} = \underbrace{\frac{\beta}{\gamma}}_{Lobbying\ channel} \underbrace{\left(\frac{P_H}{P_L}\right)^{\frac{1}{\alpha}}}_{Price\ channel} \underbrace{\left(\frac{H}{L}\right)}_{Market-size\ channel}. \tag{23}$$

Thus, considering (15), the equilibrium expression for the relative profitability of developing technologies that complement the H-sector can be:

$$\frac{V_H}{V_L} = \frac{\pi_H}{\pi_L} = \left[\left(\frac{\chi_H}{\chi_L}\right)^{\varepsilon}\left(\frac{H}{L}\right)^{\Omega}\left(\frac{N_H}{N_L}\right)^{-1}\right]^{\frac{1}{1+\Omega}}, \tag{24}$$

or

$$\frac{V_H^{\mathcal{L}}}{V_L^{\mathcal{L}}} = \frac{\pi_H^{\mathcal{L}}}{\pi_L^{\mathcal{L}}} = \frac{\beta}{\gamma}\frac{V_H}{V_L} = \frac{\beta}{\gamma}\frac{\pi_H}{\pi_L}. \tag{25}$$

Thus, $\frac{V_H}{V_L}$ depends: (i) positively on $\frac{\chi_H}{\chi_L}$ and on $\frac{H}{L}$ if $\varepsilon > 1$; (ii) negatively on $\frac{N_H}{N_L}$ and on $\frac{H}{L}$ if $\varepsilon < 1$. Additionally, $\frac{V_H^{\mathcal{L}}}{V_L^{\mathcal{L}}}$ relies positively on the interplay between firms in both sectors, $\frac{\beta}{\gamma}$, and on $\frac{V_H}{V_L}$. Along the BGP, the relative profitability $\frac{V_H(t)}{V_L(t)} = \frac{\pi_H(t)}{\pi_L(t)}$ or $\frac{V_H^{\mathcal{L}}}{V_L^{\mathcal{L}}} = \frac{\pi_H^{\mathcal{L}}}{\pi_L^{\mathcal{L}}}$, respectively, without or with the interplay between firms in both sectors in (22) or (23) is equal to relative R&D cost, which from (19) is $\frac{Z_H(t)}{Z_L(t)} = \frac{\sigma_L \cdot L^{-\delta}}{\sigma_H \cdot H^{-\delta}}$; i.e., balanced growth (steady state) technology market clearing condition implies, for example, without lobbying activity, that $\pi_H(t) \cdot \sigma_H \cdot H^{-\delta} = \pi_L(t) \cdot \sigma_L \cdot L^{-\delta}$ or, which is equivalent, $V_H(t) \cdot \sigma_H \cdot H^{-\delta} = V_L(t) \cdot \sigma_L \cdot L^{-\delta}$, resulting in the endogenous equilibrium technological-knowledge bias between the H-sector and the L-sector:

$$\mathcal{N} \equiv \frac{N_H}{N_L} = \left(\frac{\chi_H}{\chi_L}\right)^{\varepsilon}\left(\frac{\sigma_H}{\sigma_L}\right)^{1+\Omega}\left(\frac{H}{L}\right)^{\Omega(1-\delta)-\delta}, \tag{26}$$

or

$$\mathcal{N}^{\mathcal{L}} \equiv \frac{N_H^{\mathcal{L}}}{N_L^{\mathcal{L}}} = \left(\frac{\chi_H}{\chi_L}\right)^{\varepsilon}\left(\frac{\beta}{\gamma}\frac{\sigma_H}{\sigma_L}\right)^{1+\Omega}\left(\frac{H}{L}\right)^{\Omega(1-\delta)-\delta}, \tag{27}$$

which, depending on the scenario (without or with the interplay between firms in both sectors), is the key result of the directed technical change literature.[7]

From (26), regardless of the size of the elasticity of substitution between sectors in the production of the aggregate final good, ε, $\frac{N_H}{N_L}$ is positively related with $\frac{\chi_H}{\chi_L}$ and $\frac{\sigma_H}{\sigma_L}$ because $\varepsilon \geq 0$ and $1+\Omega > 0$. Moreover, the sign and intensity of the relationship between $\frac{N_H}{N_L}$ and $\frac{H}{L}$ relies on the sign and value of the exponent $\Omega(1-\delta)-\delta = (\varepsilon-1)(1-\delta)\alpha-\delta$ and, as a result, on the value of the parameters ε and δ. In turn, from (27), $\frac{N_H^{\mathcal{L}}}{N_L^{\mathcal{L}}}$ depends positively on the interplay between

[7] In the standard directed technical change literature (e.g., Acemoglu 1998, 2002, 2008), the scale has no impact on R&D technology; i.e., scale effects are not removed, $\delta = 0$, and the chain of effects is dominated by the market-size channel, by which technologies that use the more abundant labour type are favoured; thus, this literature has been interpreting the rise in the skill premium as a result of the market-size effect. In our case, however, the level of scale effects removal lends much more flexibility to the technological-knowledge bias.

firms in both sectors, $\frac{\beta}{\gamma}$, and on $\frac{N_H}{N_L}$.

That is, under substitutability, if $\varepsilon > 1$ (i.e., factors used in the two intermediate sectors are gross substitutes) and $\delta = 0$ (i.e., full scale effects on growth), the technological-knowledge bias is positively related with $\frac{H}{L}$ since the exponent is positive. In this case, such as in Acemoglu (1998, 2002, 2008), the technological-knowledge change favours the labour type employed in the larger sector of the economy due to the market-size effect, and thus technologies that use the more abundant type of labour are favoured. The idea is that the same economic forces (profitability of the R&D) that affect the technological-knowledge progress also shape the technological-knowledge bias, and the labour level is connected to the size of profits that, in each period, accrue to the leader producer – see (12); i.e., the market size affects the monopolist's profits and thus the incentives to allocate resources to R&D, thereby directing technological knowledge.[8]

Still under substitutability, if $\varepsilon > 1$ and $0 < \delta < 1$, the exponent $\Omega(1-\delta)-\delta$ is positive and the market-size channel operates when $\varepsilon > \frac{\delta+\alpha(1-\delta)}{\alpha(1-\delta)}$. In turn, when $\varepsilon = \frac{\delta+\alpha(1-\delta)}{\alpha(1-\delta)}$ the exponent is null and the technological -knowledge bias is independent of the relative supply of labour $\frac{H}{L}$. Otherwise, when $1 < \varepsilon < \frac{\delta+\alpha(1-\delta)}{\alpha(1-\delta)}$ the exponent is negative and the chain of effects is induced by the price channel by which there are stronger incentives to improve technologies when the goods that they produce command higher prices; i.e., an increase in the relative supply of labour in the H-sector biases the technological knowledge in favour of the L-sector or, in other words, technologies that use the scarcer labour are favoured.

If $\varepsilon = 1$ and $\delta = 0$, the technological-knowledge bias is also independent of of the relative supply of labour $\frac{H}{L}$.

In all other cases, the exponent $\Omega(1-\delta)-\delta$ is negative so that the price channel dominates the chain of effects.[9]

[8]The effect of the market-size channel is stronger, $\alpha(\varepsilon-1) > 1$, under intense substitutability, $\varepsilon > \frac{1+\alpha}{\alpha}$, and is directly proportional to $\frac{H}{L}$, $\Omega \equiv \alpha(\varepsilon-1) = 1$, when $\varepsilon = \frac{1+\alpha}{\alpha}$.

[9]However, the intensity of the effect depends on the case: (i) it is smaller; i.e., $-1 < \Omega(1-\delta)-\delta < 0$, when $\delta = 0$ and $\varepsilon = 0$, or $\delta = 0$ and $0 < \varepsilon < 1$, or $0 < \delta < 1$ and $\varepsilon = 0$, or $0 < \delta < 1$ and $0 < \varepsilon < 1$, or $0 < \delta < 1$ and $\varepsilon = 1$; (ii) it is inversely proportional to $\frac{H}{L}$; i.e., $\Omega(1-\delta)-\delta = -1$, whenever $\delta = 1$; and (iii) it is stronger; i.e., $\Omega(1-\delta)-\delta > -1$, whenever $\delta > 1$.

2.3 Lobbying implications

We model the interaction between firms producing intermediate goods in both sectors, H and L, as a kind of game.[10] According to the above analysis, the payoff to intermediate goods firms depends on the demand from producers of final goods, and the demand relies on the path (progress and bias) of the technological knowledge, which, in turn, is affected by the lobbying activity. That is, if the intermediate-goods firms in the H-sector practice the price β due to the lobbying activity in order to regulate the path of the technological knowledge, the intermediate-goods firms in the L-sector can practice the price γ with the same purpose, knowing that the lobbying power differs between sectors.

In particular, since the path of the technological knowledge determines the dynamics of the economy and the path of the technological knowledge depends on the interaction between firms in both sectors, the interplay between firms should take into account the equilibrium expression for the relative profitability of the intermediate-goods firms in the H-sector, (12) and (13). Moreover, we consider a scenario wherein the lobbying power of the intermediate-goods firms in the H-sector differs from the lobbying power of intermediate-goods firms in the L-sector, by assuming that the exogenously given parameter ϕ evaluates the relative lobbying power of the intermediate-goods firms in the H-sector.Thus, the relative profitability of the intermediate-goods firms in the H-sector is:

$$H - firms : \begin{cases} \frac{\pi_H}{\pi_L} - \beta & if\ \phi\beta \geq \gamma \\ \frac{\pi_H^{\mathcal{L}}}{\pi_L^{\mathcal{L}}}\phi - \beta & if\ \phi\beta < \gamma \end{cases}, \tag{28}$$

for a scenario in which the relative profitability of the intermediate-goods firms corresponds to the baseline one without lobbying, if $\phi\beta \geq \gamma$, or with lobbying, if $\phi\beta < \gamma$, considering that β represents the relative lobbying cost of the intermediate-goods firms in the H-sector, which simplifies the notation without any loss of generality. Thus, the relative profitability of the intermediate-goods firms in the L-sector is:

[10] We follow the idea of Le Breton et al. (2012), who employ this type of set-up in a model of legislative lobbying.

$$L - firms : \begin{cases} \left(1 - \frac{\pi_H}{\pi_L}\right) - \gamma & if\ \phi\beta \geq \gamma \\ \left(1 - \frac{\pi_H^{\mathcal{L}}}{\pi_L^{\mathcal{L}}}\phi\right) - \gamma & if\ \phi\beta < \gamma \end{cases} , \tag{29}$$

for a scenario in which the relative profitability of the respective firms corresponds to the baseline without lobbying, if $\phi\beta \geq \gamma$, or with lobbying, if $\phi\beta < \gamma$, and γ measures the relative lobbying cost of the intermediate-goods firms in the L-sector, which simplifies the notation without any loss of generality.

To solve the problem we use backward induction wherein the intermediate-goods firms in the L-sector decide on the optimal γ for a given β. Then, the intermediate-goods firms in the H-sector use this information to decide on their optimal share of lobbying resources. From (29), in the interval $[0, \phi\beta]$ the relative profitability of the intermediate-goods firms in the L-sector decreases in γ and thus the optimal γ is 0. Therefore, for an interior solution with $\gamma > 0$, $\gamma \in (\phi\beta, 1)$ and in order that the intermediate-goods firms in the L-sector expend an amount $\gamma > \phi\beta$ the first derivative of the respective expression with respect to γ evaluated at $\phi\beta$ must be positive, which requires that:

$$\beta < \frac{\pi_H}{\pi_L}\frac{1}{\phi} \equiv \tilde{\beta}. \tag{30}$$

Assuming this condition holds, one finds that the relative profitability of the intermediate-goods firms in the L-sector in the interval $(\phi\beta, 1)$, $\left(1 - \frac{\pi_H^{\mathcal{L}}}{\pi_L^{\mathcal{L}}}\phi\right) - \gamma$, is maximized at

$$\gamma = \left(\frac{\pi_H}{\pi_L}\phi\beta\right)^{\frac{1}{2}}. \tag{31}$$

However, this value of γ is optimal if and only if the relative profitability at γ given by (31) is greater than what is obtained when $\gamma = 0$, which requires that:

$$\beta < \frac{\pi_H}{\pi_L}\frac{1}{4\phi} \equiv \hat{\beta}. \tag{32}$$

Hence, if (32) is satisfied then (30) also holds, since $\hat{\beta} > \tilde{\beta}$, which establishes that the optimal value for γ is

$$\gamma^* = 0,\ if\ \beta \geq \hat{\beta} \equiv \frac{\pi_H}{\pi_L}\frac{1}{4\phi}\ ;\ \gamma^* = \left(\frac{\pi_H}{\pi_L}\phi\beta\right)^{\frac{1}{2}},\ if\ \beta < \hat{\beta} \equiv \frac{\pi_H}{\pi_L}\frac{1}{4\phi} \tag{33}$$

Bearing in mind the optimal values for γ in (33), we can now find the optimal share of resources expended by the intermediate-goods firms in the H-sector in lobbying, $\beta^* \in (0,1)$, which, as is apparent from (33), lies in the interval $[0,\hat{\beta}]$. From (33), the relative profitability of R&D H-firms is

$$H - firms: \begin{cases} \frac{\pi_H}{\pi_L} - \hat{\beta} & if\ \beta = \hat{\beta} \\ \left(\frac{\pi_H}{\pi_L}\phi\beta\right)^{\frac{1}{2}} - \beta & if\ \beta < \hat{\beta} \end{cases} . \tag{34}$$

Proposition 1. *The relative profitability of the intermediate-goods firms in the H-sector depends on the relative lobbying strength of the intermediate-goods firms in the H-sector, ϕ: (i) if ϕ is low, $\phi \leq 0.268$, then the intermediate-goods firms in the L-sector have the advantage in lobbying; (ii) if ϕ is big enough, $\phi > 0.268$, then the intermediate-goods firms in the H-sector have the advantage in lobbying. Regardless of the case, (i) or (ii), when ϕ increases then the relative profitability of the intermediate-goods firms in the H-sector also increases.*

Proof. From (34), for $\beta < \hat{\beta}$ the first derivative of the respective expression with respect to β, $\frac{\partial\left[\left(\frac{\pi_H}{\pi_L}\phi\beta\right)^{\frac{1}{2}} - \beta\right]}{\partial\beta} = \left(\frac{\pi_H}{\pi_L}\frac{\phi}{\beta}\right)^{\frac{1}{2}}\frac{1}{2} - 1$, is positive, negative, or null as $\beta < \beta^* = \frac{\pi_H}{\pi_L}\frac{\phi}{4}$, $\beta > \beta^*$, or $\beta = \beta^*$; hence, the relative profitability of the intermediate-goods firms in the H-sector is maximized at $\hat{\beta}$ if $\hat{\beta} \equiv \frac{\pi_H}{\pi_L}\frac{1}{4\phi} \leq \beta^* = \frac{\pi_H}{\pi_L}\frac{\phi}{4}$, that is, if $\phi \geq 1$. In turn, if $\phi < 1$ then $\beta^* < \hat{\beta}$ and $\left(\frac{\pi_H}{\pi_L}\frac{\phi}{\beta}\right)^{\frac{1}{2}}\frac{1}{2} - 1 = 0$ at $\beta^* \in [0,\hat{\beta})$, which is the optimal value for β; in this case,

$$\frac{\phi\beta^*}{\gamma^*} = \frac{\phi}{2}, \tag{35}$$

i.e., $\frac{\phi\beta^*}{\gamma^*} < 1$, and the relative profitability of the intermediate-goods firms in the H-sector is $\left(\frac{\pi_H}{\pi_L}\phi\frac{\pi_H}{\pi_L}\frac{\phi}{4}\right)^{\frac{1}{2}} - \frac{\pi_H}{\pi_L}\frac{\phi}{4} = \frac{\pi_H}{\pi_L}\frac{\phi}{4}$ when $\beta = \beta^*$and is $\frac{\pi_H}{\pi_L}\left(1 - \frac{1}{4\phi}\right)$ when $\beta = \hat{\beta}$. Thus, when $\beta^* < \hat{\beta}$, we conclude that $\frac{\pi_H}{\pi_L}\frac{\phi}{4} - \frac{\pi_H}{\pi_L}\left(1 - \frac{1}{4\phi}\right) > 0, < 0, = 0$ if $\phi^2 - 4\phi + 1 > 0,\ < 0,\ = 0$, which has zeros at $\phi = 2 - 3^{\frac{1}{2}} = 0.268$ and $\phi = 2 + 3^{\frac{1}{2}} = 3.732$ and is negative in the interval between zeros. Hence, when

(i) $\phi \leq 0.268$, the optimal value for β is $\beta^* = \frac{\pi_H}{\pi_L}\frac{\phi}{4}$, $\gamma^* = \left(\frac{\pi_H}{\pi_L}\phi\beta^*\right)^{\frac{1}{2}}$ and there is an interior solution with $\frac{\phi}{2} < 1$, and the relative profitability of the intermediate-goods firms in the H-sector is $\frac{\pi_H}{\pi_L}\frac{\phi}{4}$, which depends positively on ϕ;

(ii) $0.268 < \phi < 3.732$ as well as when $\phi \geq 3.732$, the optimal value for β is $\beta^* = \hat{\beta} = \frac{\pi_H}{\pi_L}\frac{1}{4\phi}$, $\gamma^* = 0$, and the relative profitability of the intermediate-goods firms in the H-sector is $\frac{\pi_H}{\pi_L}\left(1-\frac{1}{4\phi}\right)$, which depends positively on ϕ.

□

Now, the aim is to find the endogenous equilibrium technological-knowledge bias between the H-sector and the L-sector. Along the BGP, the relative R&D cost $\frac{Z_H(t)}{Z_L(t)} = \frac{\sigma_L \cdot L^{-\delta}}{\sigma_H \cdot H^{-\delta}}$ is equal to the relative profitability of R&D H-firms, which now is

- $\frac{\pi_H}{\pi_L}\frac{\phi}{4}$ when the intermediate-goods firms in the L-sector have the lobbying advantage; i.e., when $\phi \leq 0.268$.
- $\frac{\pi_H}{\pi_L}\left(1-\frac{1}{4\phi}\right)$ when the intermediate-goods firms in the H-sector have the lobbying advantage; i.e., when $\phi > 0.268$.

That is, $\frac{Z_H(t)}{Z_L(t)} = \frac{\sigma_L \cdot L^{-\delta}}{\sigma_H \cdot H^{-\delta}} = \begin{cases} \frac{\pi_H}{\pi_L}\frac{\phi}{4} & if\ \phi \leq 0.268 \\ \frac{\pi_H}{\pi_L}\left(1-\frac{1}{4\phi}\right) & if\ \phi > 0.268 \end{cases}$. Thus, considering (24), the new technological-knowledge bias, $\mathcal{N}_1$, is:

$$\mathcal{N}_1 = \mathcal{N} \times \begin{cases} \left(\frac{\phi}{4}\right)^{1+\Omega} & if\ \phi \leq 0.268 \\ \left(1-\frac{1}{4\phi}\right)^{1+\Omega} & if\ \phi > 0.268 \end{cases}. \tag{36}$$

The following theorem presents the technological-knowledge bias with lobbying activity.

Theorem 1. *Due to lobbying, the technological-knowledge bias, $\mathcal{N}_1$, becomes less biased toward the H-sector. However, as expected, the difference between $\mathcal{N}$ and $\mathcal{N}_1$ is greater when the intermediate-goods firms in the L-sector have the lobbying advantage than when the intermediate-goods firms in the H-sector have the lobbying advantage.*

Proof. Since $1+\Omega > 1$, it results directly from (36); indeed,

$$\lim_{\phi \to 0.268}\left(\frac{\phi}{4}\right) = 0.067; \tag{37}$$

$$\lim_{\phi \to 0.268}\left(1-\frac{1}{4\phi}\right) = 0.067,\ \lim_{\phi \to +\infty}\left(1-\frac{1}{4\phi}\right) = 1. \tag{38}$$

$\square$

As a result of the adjustment of the technological-knowledge bias in (36), the relative profitability of developing technologies that complement the H-sector, $\frac{V_H}{V_L}$, in (22) becomes given by

$$\frac{V_H}{V_L} = \left(\frac{\sigma_H}{\sigma_L}\right)^{-1}\left(\frac{H}{L}\right)^{\delta} \times \begin{cases} \left(\frac{\phi}{4}\right)^{-1} & if\ \phi \leq 0.268 \\ \left(1-\frac{1}{4\phi}\right)^{-1} & if\ \phi > 0.268 \end{cases} . \tag{39}$$

and, thus, it depends: (i) negatively on the relative productivity of developing H-sector complementary intermediate goods, $\frac{\sigma_H}{\sigma_L}$, and on the lobbying strength, ϕ – and, thus, lobbying acts in the free-entry condition;[11] (ii) positively on relative supply of labour, $\frac{H}{L}$.

We are also interested in obtaining the expression for the relative price, $\frac{P_H}{P_L}$, with lobbying along the BGP. From (15) and due to complementarity between inputs in (3), the relative price of the H-sector is negatively related with the endogenous technological-knowledge bias, $\mathcal{N}_1$. Thus, substituting (36) into (15), along the BGP, after the adjustment of the technological-knowledge bias, the price ratio is:

$$\frac{P_H}{P_L} = \left(\frac{\sigma_H}{\sigma_L}\right)^{-\alpha}\left(\frac{H}{L}\right)^{\frac{-\alpha-\alpha[(\varepsilon-1)(1-\delta)\alpha-\delta]}{1+\Omega}} \times \begin{cases} \left(\frac{\phi}{4}\right)^{-\alpha} & if\ \phi \leq 0.268 \\ \left(1-\frac{1}{4\phi}\right)^{-\alpha} & if\ \phi > 0.268 \end{cases} . \tag{40}$$

Moreover, we analyse the implications of endogenous technological-knowledge bias on the relative wage, $\frac{w_H}{w_L}$, since, by reason of complementarity between factors in (3), changes in $\frac{w_H}{w_L}$ are closely related to the technological-knowledge bias, as (16) clearly shows. Hence, we substitute (36) into (16):

$$\frac{w_H}{w_L} = \left(\frac{\chi_H}{\chi_L}\right)^{\varepsilon}\left(\frac{\sigma_H}{\sigma_L}\right)^{\Omega}\left(\frac{H}{L}\right)^{\Omega(1-\delta)-1} \times \begin{cases} \left(\frac{\phi}{4}\right)^{\Omega} & if\ \phi \leq 0.268 \\ \left(1-\frac{1}{4\phi}\right)^{\Omega} & if\ \phi > 0.268 \end{cases} . \tag{41}$$

Finally, to discuss in further detail the impact of ϕ in the wage ratio, we calculate the partial derivative in order to the relative lobbying power of the

[11] Lobbying therefore takes place over the allocation of intermediate-goods resources between sectors, H and L.

leader H-firm. The resulting expression is:

$$\frac{\partial\left(\frac{w_H}{w_L}\right)}{\partial\phi}=\left(\frac{\chi_H}{\chi_L}\right)^{\varepsilon}\left(\frac{\sigma_H}{\sigma_L}\right)^{\Omega}\left(\frac{H}{L}\right)^{\Omega(1-\delta)-1}\frac{\Omega}{4}\times\begin{cases}\left(\frac{\phi}{4}\right)^{\Omega-1} & if\ \phi\leq 0.268\\ \left(1-\frac{1}{4\phi}\right)^{\Omega-1}\frac{1}{\phi^2} & if\ \phi>0.268\end{cases}. \tag{42}$$

Proposition 2. *The elasticity of the wage premium, $\frac{w_H}{w_L}$, in relation to changes in the relative lobbying power of the intermediate-goods firms in the H-sector, ϕ, depends crucially on the elasticity of substitution. Indeed, $\frac{\partial\left(\frac{w_H}{w_L}\right)}{\partial\phi}$ is*

A. positive if $\epsilon>1$ ($\Omega>0$) or negative if $\epsilon<1$ ($\Omega<0$).

B. (i) positively, null, or negatively related with the relative lobbying power of the intermediate-goods firms in the H-sector, ϕ, when the elasticity of substitution is greater than, equal to, or less than $\frac{1+\alpha}{\alpha}$ in a context with the intermediate-goods firms in the L-sector lobbying advantage; i.e., when $\phi\leq 0.268$; (ii) negatively related with the relative lobbying power of the intermediate-goods firms in the H-sector, ϕ, in a context with the intermediate-goods firms in the H-sector lobbying advantage; i.e., when $\phi>0.268$.

C. (i) positively related with: (a) the relative importance of the H-sector in the production of the final good, $\frac{\chi_H}{\chi_L}$; (b) both the relative productivity of developing H-sector complementary intermediate goods, $\frac{\sigma_H}{\sigma_L}$, and the share of labour in production, α (through Ω), but just when $\varepsilon>1$; (ii) positively, null, or negatively related with the relative supply of labour in the H-sector, $\frac{H}{L}$, according the sign of the exponent $\Omega(1-\delta)-1$.

Proof. From (42), to prove A. it suffices to evaluate to the sign of $\Omega\equiv(\varepsilon-1)\alpha$ when there is substitutability ($\epsilon>1$) or complementarity ($\epsilon<1$).

The elasticity of the wage premium, $\frac{w_H}{w_L}$, in relation to changes in the relative lobbying power of the intermediate-goods firms in the H-sector, ϕ, depends crucially on Ω and thus on ε, in a context with the intermediate-goods firms in the H-sector lobbying advantage; i.e., when $\phi\leq 0.268$. Indeed, when ε is greater than, equal to, or less than $\frac{1+\alpha}{\alpha}$, the respective exponent, $\Omega-1$, is greater than, equal to, or less than 0. Thus, in this case, the elasticity $\frac{\partial\left(\frac{w_H}{w_L}\right)}{\partial\phi}$ is positively, null, or negatively related with ϕ. In turn, the elasticity $\frac{\partial\left(\frac{w_H}{w_L}\right)}{\partial\phi}$

relies negatively on ϕ when the intermediate-goods firms in the H-sector have lobbying advantage; i.e., when $\phi > 0.268$.

Moreover, the elasticity of the wage premium, $\frac{w_H}{w_L}$, in relation to changes in the relative lobbying power of the intermediate-goods firms in the H-sector, ϕ, is always positively related with the relative importance of the H-sector in the production of the final good, $\frac{\chi_H}{\chi_L}$, since $\varepsilon > 0$, as well as with the relative productivity of developing H-sector complementary intermediate goods, $\frac{\sigma_H}{\sigma_L}$, when $\varepsilon > 1$ since, in this case, $\Omega > 0$. Through the term $\frac{\Omega}{4}$, it also depends positively on the share of labour in production, α, when $\varepsilon > 1$.

However, the sign of the effect of $\frac{H}{L}$ on $\frac{w_H}{w_L}$ is ambiguous, as is shown in Table 3. That is, under substitutability, if (i) $\frac{1+\alpha}{\alpha} < \varepsilon < +\infty$ and $\delta = 0$ or (ii) $\frac{1+\alpha(1-\delta)}{\alpha(1-\delta)} < \varepsilon < +\infty$ and $0 < \delta < 1$, as well as under complementarity, if (iii) $\varepsilon = 0$ and $\delta > \frac{1+\alpha}{\alpha}$ or (iv) $0 < \varepsilon < 1$ and $1 < \delta < \frac{(\varepsilon-1)\alpha-1}{(\varepsilon-1)\alpha}$, an increase of $\frac{H}{L}$ increases $\frac{w_H}{w_L}$ since the effect of $\frac{H}{L}$ on $\frac{N_H}{N_L}$, which drives wage-inequality dynamics, dominating the direct effect induced by the relative supply.[12] In these four cases, the increase in the supply of $\frac{H}{L}$, causes, due to the usual substitution effect – see (16) – an immediate steep drop in $\frac{w_H}{w_L}$ since the relative labour supply decreases the respective relative wage. This immediate effect is however reversed in the transitional dynamics toward the new constant steady state of $\frac{w_H}{w_L}$, due to the stimulus to the demand for $\frac{H}{L}$ resulting from the induced $\mathcal{N}_1$ in favour of the H-sector. Once in steady state, with a constant $\mathcal{N}_1$, $\frac{w_H}{w_L}$ remains also constant.

Under sustitutability, if $\varepsilon = \frac{1+\alpha}{\alpha}$ and $\delta = 0$ or $\varepsilon = \frac{1+\alpha(1-\delta)}{\alpha(1-\delta)}$ and $0 < \delta < 1$, as well as under complementarity, if $\varepsilon = 0$ and $\delta = \frac{1+\alpha}{\alpha}$ or $0 < \varepsilon < 1$ and $\delta = \frac{1+\alpha(1-\delta)}{\alpha(1-\delta)}$, $\frac{w_H}{w_L}$ is independent of $\frac{H}{L}$.

In all other cases, the exponent $\alpha(\varepsilon-1)(1-\delta)-1$ is always negative so that the price channel dominates the chain of effects, such that in view of, for example, an increase in the relative supply of H, $\frac{H}{L}$, the relative price of the H-sector decreases, $\frac{P_H}{P_L}$, which discourages R&D activities in this sector, biasing the technological-knowledge in favour of the L-sector and thus generates a decrease of $\frac{w_H}{w_L}$. However, the intensity of the effect depends on the case: (i) it is smaller; i.e., $-1 < \alpha(\varepsilon-1)(1-\delta)-1 < 0$, when $1 < \varepsilon < \frac{1+\alpha}{\alpha}$ and $\delta = 0$, or $1 < \varepsilon < \frac{1+\alpha(1-\delta)}{\alpha(1-\delta)}$ and $0 < \delta < 1$, or $\varepsilon = 0$ and $\delta < \frac{1+\alpha}{\alpha}$,

[12] Comparing (41) to (16) results that, in line with the *LeChatelier* principle, the response of relative wages $\frac{w_H}{w_L}$ to changes in relative supply $\frac{H}{L}$ is now more elastic since the respective demand curves become more elastic after the adjustment of the "other factors", which here correspond to the number of intermediate goods N_H and N_L; thus, bearing in mind the exponents of $\frac{H}{L}$ it results that $\alpha(\varepsilon-1)(1-\delta)-1 > -\frac{1}{\alpha(\varepsilon-1)-1}$ in all these cases.

	$\delta = 0$	$0 < \delta < 1$	$\delta = 1$	$\delta > 1$
$\varepsilon = 0$	< -1	< -1	$= -1$	$(-\infty, +\infty)$
$0 < \varepsilon < 1$	< -1	< -1	$= -1$	$(-\infty, +\infty)$
$\varepsilon = 1$	$= -1$	$= -1$	$= -1$	$= -1$
$\varepsilon > 1$	$(-\infty, +\infty)$	$(-\infty, +\infty)$	$= -1$	< -1

Table 1: Elasticity of the relative wage, $\frac{w_H}{w_L}$, with respect to the labour ratio, $\frac{H}{L}$.

or $0 < \varepsilon < 1$ and $\delta > \frac{(\varepsilon-1)\alpha-1}{(\varepsilon-1)\alpha}$; (ii) it is inversely proportional to $\frac{H}{L}$; i.e., $\alpha(\varepsilon-1)(1-\delta)-1 = -1$, when $\varepsilon = 1$ ($\delta = 1$) whatever the value of δ (ε); and (iii) it is stronger; i.e., $\alpha(\varepsilon-1)(1-\delta)-1 < -1$, when $\varepsilon = 0$ and $\delta = 0$, or $\varepsilon = 0$ and $0 < \delta < 1$, or $0 < \varepsilon < 1$ and $\delta = 0$, or $0 < \varepsilon < 1$ and $0 < \delta < 1$, or $\varepsilon > 1$ and $\delta > 1$.

[Table 1 goes about here].

□

To sum up, bearing in mind the misallocation of resources that affect the direction of technological-knowledge change, which, in turn, affect the relative demand of high-skilled labour and the skill premium, we are able to accommodate the distinct cross-country paths of both the skill premium and the relative supply of workers. The technological-knowledge bias is affected by six factors: (i) the elasticity of substitution between the two final goods; (ii) the relative labour levels of both goods; (iii) the relative importance of both goods in the production of the unique final good; (iv) the relative advantage to entry through horizontal innovation in one sector; (v) market complexity effect in R&D activities; (vi) allocative inefficiencies due to lobbying by firms. Due to lobbying, the technological-knowledge bias becomes less biased toward the skilled sector, thus producing a "wrong" mix of goods and a lower skill premium in the long run. However, when the relative lobbying strength of the intermediate-goods firms in the H-sector increases, the relative profitability of R&D in the H-sector also increases and, consequently, so does the (lower) skill premium under substitutability between sectors. The elasticity of the wage premium in relation to changes in the relative lobbying power of the intermediate-goods firms in the H-sector depends positively, null, or negatively on the relative lobbying power of the intermediate-goods firms in the H-sector when the elasticity of substitution is higher than the threshold $\frac{1+\alpha}{\alpha}$ in a context with the intermediate-goods firms in the L-sector having the lobbying advantage, whereas it relies negatively on the relative lobbying power of the intermediate-goods firms in the H-sector in a context with the intermediate-goods firms in the H-sector lobbying advantage.

To close the model we need to find the long-run economic growth rate of the economy. Taking into account that agents are delaying consumption by investing in R&D, as a function of the interest rate, and bearing in mind (2), in BGP, the consumption, C, the total investment in production of intermediate goods, X, the aggregate R&D expenditures, Z, the aggregate output, Y, and the technological knowledge grow at the same rate, g; i.e., C, X, Z, and Y are multiples of N_H and N_L. Thus, since the country's interest rate is unique, from (1) $r = \theta g + \rho$ and this condition ensures that the steady-state growth rate is unique. Hence, along the BGP, the growth rate of the economy depends on the market interest rate and preference parameters. Now, using (21), the free-entry condition for the technology monopolists working to invent j-complementary intermediate goods implies that $V_j = Z_j$, which is equivalent to stating that $L_j^{-\delta}\sigma_j V_j = 1$ since $W_j^{-\delta}\sigma_j Z_j$ units of composite final-good resources are required to invent a new design (to produce a new intermediate good); that is, $W_j^{-\delta}\sigma_j Z_j = 1$. In steady state, this condition implies that $W_j^{-\delta}\sigma_j V_j = \frac{W_j^{-\delta}\sigma_j \cdot \alpha \cdot P_j^{1/\alpha} \cdot W_j}{r} = 1$, which when solved, for example, for $j = L$ implies $r = L^{1-\delta} \cdot \sigma_j \cdot \alpha \cdot P_L^{1/\alpha}$. Then, using (15) and (36), we obtain an expression for P_L, which together with (4) allows us to determine the steady-state interest rate dependent only on exogenous parameters and variables. Finally, replacing the steady-state interest rate in the Euler equation (1), we find the steady-state economic growth rate:

$$g = \frac{1}{\theta} \times \begin{cases} \alpha \left(\frac{\phi}{4}\right)^{\Omega} \left[\chi_H^{\varepsilon} \left(\sigma_H H^{1-\delta}\right)^{\Omega} + \chi_L^{\varepsilon} \left(\sigma_L L^{1-\delta}\right)^{\Omega}\right]^{\frac{1}{\Omega}} - \rho & if\ \phi \leq 0.268 \\ \alpha \left(1 - \frac{1}{4\phi}\right)^{\Omega} \left[\chi_H^{\varepsilon} \left(\sigma_H H^{1-\delta}\right)^{\Omega} + \chi_L^{\varepsilon} \left(\sigma_L L^{1-\delta}\right)^{\Omega}\right]^{\frac{1}{\Omega}} - \rho & if\ \phi > 0.268 \end{cases}. \tag{43}$$

from which higher productivity in R&D activity, σ_j, makes the steady-state growth rate of the economy higher, while the rate of time preference, ρ, and the elasticity of marginal utility of consumption, θ, have depressing effects on the steady-state growth rate: the impact of ρ and θ on g is consistent with the fact that if present consumption is more highly valued than future consumption (following the properties of the utility function), then this will lead to less need for private investment and so to less dispersion over time; that is, the more patient – i.e., the smaller the value of ρ – and the less keen the individuals are on consumption smoothing – i.e., the smaller the value of θ – the higher the steady-state growth rate. Moreover, it is clear from (43) that indeed scale effects can be present ($\delta = 0$), affecting positively the economic growth rate, can be partially removed ($0 < \delta < 1$), having a smaller impact on economic growth, can be totally removed ($\delta = 1$), no longer affecting the economic growth rate, or can over counterbalance the scale benefits on profits ($\delta > 1$) and thus affect negatively the economic growth rate. Finally, the impact of lobbying activity on the steady-state growth rate should be stressed.

The impact of the relative lobbying power of the intermediate-goods firms in the H-sector, ϕ, on the steady-state growth rate depends on the value of ε in (3); i.e., under substitutability, $\varepsilon > 1$, $\frac{\partial g}{\partial \phi} < 0$, while under complementarity, $\varepsilon < 1$, $\frac{\partial g}{\partial \phi} > 0$. When the unskilled-final-goods sector, Y_L, and the skilled-final-goods sector, Y_H, are substitutable in (3), it is possible to replace the less efficient productive sector, Y_L, with the more efficient, Y_H, promoting the growth rate of Y_H; however, an increase in the relative lobbying strength of the intermediate-goods firms in the H-sector disrupts the correct allocation and, therefore, penalizes the economic growth rate. Moreover, as the terms $\left(\frac{\phi}{4}\right)^{\Omega}$ and $\left(1 - \frac{1}{4\phi}\right)^{\Omega}$ in (43) are greater than 1 under complementarity, $\varepsilon < 1$, and smaller than 1 under substitutability, $\varepsilon > 1$, the model accommodates both "growth miracles" (high-growth equilibrium), namely when $\varepsilon \to 0$, and "poverty traps" (low-growth equilibrium), namely when $\varepsilon \to +\infty$.

Finally, it should be also emphasised that the interior steady state is stable since outside the BGP, just one type of innovation arises; i.e., innovations for the L-sector or for the H-sector:[13] considering, as example, the case without lobbying activity, when $\frac{V_H(t)}{V_L(t)} > \frac{\sigma_L L^{-\delta}}{\sigma_H H^{-\delta}}$ R&D activity is directed to create new designs to produce H-complementary intermediate goods and when $\frac{V_H(t)}{V_L(t)} < \frac{\sigma_L L^{-\delta}}{\sigma_H H^{-\delta}}$ R&D activity is directed to create new designs to produce L-complementary intermediate goods. Moreover, as, for example, (24) shows, $\frac{V_H}{V_L}$ is decreasing in $\frac{N_H}{N_L}$, which implies that the transitional dynamics of the system are stable and the economy always returns to the BGP. When $\frac{N_H}{N_L}$ is higher (low) than in (36), the technological-knowledge change will only be directed to L-complementary (H-complementary) intermediate goods until the economy returns back to the BGP in which both sectors grow at the same rate, whereas when $\frac{N_H}{N_L}$ is lower than in (36), the technological-knowledge change will only be directed to H-complementary intermediate goods until the economy returns back to the BGP in which both sectors grow at the same rate. Thus, without

[13]See proposition 1 in Acemoglu and Zilibotti (2001) for a formal poof that only innovation for the L-sector or the H-sector will occur outside the BGP, as well as for a proof of global stability in a closed related model; alternatively, see Acemoglu (2002).

any exogenous disturbance, the economy converges and remains in the unique and stable steady state in which both sectors grow at the same rate.

3 Calibration and quantitative implications

In this Section we calibrate the model and evaluate the quantitative implications of the relative lobbying power to wage inequality (in this DTC model, equal to the skill-premium) and economic growth. We note that there are few previous attempts to take to data a DTC growth model as it is difficult to distinguish between skilled and unskilled sectors and, in addition, to distinguish between the innovations toward each of the skilled and unskilled sectors. It is to avoid some of these difficulties that we concentrate on the analysis of the wage inequality and growth.[14]

To calibrate the model we begin with the value for the elasticity of substitution between intermediate final goods (the L-sector and the H-sector), ε. Baier et al. (2012) provide evidence according to which a value of $\varepsilon = 2.03$ is more preferable than a value of $\varepsilon = 1.6$, pointing to the existence of strong biased technological-knowledge change. Duffy and Papageourgiou (2000) present estimates for ε using a panel database for 82 countries over a 28-year period. Nonlinear estimations for ε oscillate between 1.2 and 2.3, while linear estimations oscillate around 1.4. In fact, only one of the estimations for a subsample of initially poorer countries presents an elasticity of substitution less than 1 (around 0.8). It is worth noting that in estimations for initially rich countries, the elasticity of substitution is not significantly different from 1. In Goldin and Katz (2007), the estimation of the elasticity of substitution for the USA yields a value of 1.64. In a baseline exercise we will consider $\varepsilon = 2$, and in alternate exercises we consider the value 1.4; i.e., between 1 and 2, and another one close to and less than 1, 0.8. We consider a share of labour $\alpha = \frac{2}{3}$, following the stylized fact for the USA. For the ratio of R&D productivities, $\frac{\sigma_H}{\sigma_L}$ we use the value for the year 2000 in Baier et al. (2012, Figure 12), measuring the efficiency of research for College relative to High School, $\frac{\sigma_H}{\sigma_L} = 5$. The parameter δ governs the strength of the scale effects in the economy. As we are considering the cali-

[14]With this, we avoid calibrating values related to the stock of knowledge in both R&D sectors.

bration of a developed economy, we consider that the scale effect should be very small or zero – as suggested by Jones (1995a, b) and Peretto (1998). Thus, for the baseline calibration, we consider $\delta = 1$. However, as Alesina et al. (2005) also considered that the scale effects should take into consideration the openness of the countries, we also consider an alternative (intermediate) value of $\delta = 0.5$. Finally, to have quantitative effects for wage inequality – see equation (36) – we also need values for the share of the skilled sector, H-sector, and the share of unskilled sector, L-sector, in the economy. As it is quite difficult to associate sectors with the skilled levels, we used data from the ILOstat (the database of the International Labor Organization) for the USA in 2004 and classify the sector into skilled and unskilled according to the share of skilled occupations that they employ. We consider as skilled workers those included in the following classes: legislators, senior officials, and managers; professionals and technicians and associated professionals. And we consider as unskilled workers those included in the following categories: clerks; service workers and shop and market sales workers; agricultural and fishery workers; craft and related trade workers; plant and machine operators and assemblers and elementary occupations. Then, we calculate the ratio between skilled workers and unskilled workers by sector and consider as skilled sector one that has more than 0.6 in that ratio. Following this methodology, we find the skilled intensive sectors – agriculture, real-estate, finance services, public services, education, social and health services – and the unskilled intensive sectors – construction, hotels and restaurants, mining, manufacturing, retail, transports and communications, and utilities. Finally, we calculate the share of the unskilled sector and skilled sector in total GDP (according to the Bureau of Economic Analysis in the same year), which we use as a proxy for the share of each sector in the economy, giving $\chi_L = 0.43$ and $\chi_H = 0.57$. These values are all we need to calculate the skilled wage ratio or wage inequality.

3.1 Calibration of parameters

In order to calculate the economic growth rate, equation (38), we need two additional parameters, the intertemporal discount rate, ρ, and the intertemporal elasticity of substitution, θ. We use estimates from Chen et al. (2013) to calibrate these parameters, yielding $\rho = 0.0101$ and $\theta = 60$ – Table 2. Finally, values for δ_H , δ_L , H and L have now to be used separately, and thus we use

	α	ε	$\frac{\sigma_H}{\sigma_L}$	δ	χ_H	χ_L	ρ	θ
baseline	0.66	2	$\frac{10}{2}$ =5	1	0.57	0.43	0.0101	60
alternative ε	–	1.4	–	–	–	–	–	–
alternative ε	–	0.8	–	–	–	–	–	–
alternative δ	–	–	–	0.5	–	–	–	–

Table 2: Values of parameters in the calibration.

$\delta_H = 10$ and $\delta_L = 2$, respecting the above mentioned ratio of 5, which yields a reasonable economic growth rate of 1.95% with a relative lobbying strength of the intermediate-goods firms in the H-sector equal to 1, $\phi = 1$, and we use data for the employed full time workers from the Current Population Survey for the USA (data for 2015, in thousands of people) to obtain values for skilled workers (bachelor and advanced degrees), $H = 40726$, and for unskilled workers (high school and less), $L = 32632$.

3.2 Quantitative Results

Now, our main goal is to analyse the effect of different relative lobbying power of firms in the skilled wage ratio or wage inequality and in economic growth, as well as evaluate the effect of the existence of lobbying on these important macroeconomic variables. To that end we plot wage inequality toward the skilled/unskilled labour ratio, $\frac{H}{L}$, and relative lobbying power parameter, ϕ, for the four sets of parameters values inserted in Table 3. Figure 1 plots those results.

[Figure 1 goes about here]

In Figures 1a), 1b), and 1c), wage inequality increases with the relative lobbying strength of the intermediate-goods firms in the H-sector. In fact, those firms guarantee an increase in profitability with lobbying, which biases the technological-knowledge progress toward those firms, and thus influence positively the skill premium. However, the shape of the relationship between the skill premium and the relative lobbying strength of the intermediate-goods firms in the H-sector changes above the threshold point, $\phi = 0.268$, above which the H-firms have relative advantage in lobbying. As Proposition 1 states, regardless of this threshold point, the H-firms profitability increases with the relative lobbying strength of the intermediate-goods firms in the H-sector.

As Figures 1a), 1b), 1c), and 1d) also show, we can observe both a negative

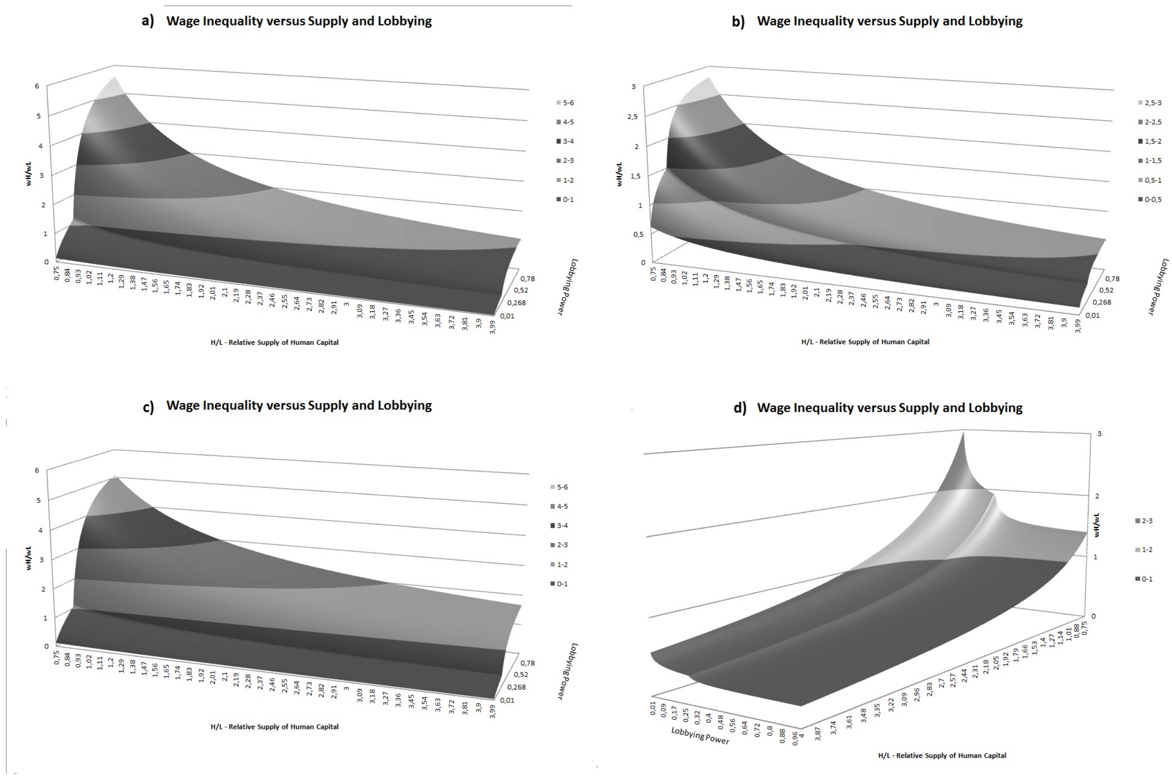

Figure 1: Wage/skill premium, $\frac{w_H}{w_L}$, for different values of the skilled/unskilled ratio, $\frac{H}{L}$, and the lobbying effort, ϕ: a) Elasticity of Substitution, $\varepsilon = 2$, and no scale effects, $\delta = 1$; b) Elasticity of Substitution, $\varepsilon = 1.4$, and no scale effects, $\delta = 1$; c) Elasticity of Substitution, $\varepsilon = 2$, and intermediate scale effects, $\delta = 0.5$; d) Elasticity of Substitution, $\varepsilon = 0.8$, and no scale effects, $\delta = 1$.

or a positive relationship between the skill premium and the relative supply of skills, depending on the relative lobbying strength of the intermediate-goods firms in the H-sector. Overall, for a constant lobbying effort, and for this elasticity of substitution and no scale effects, there is a negative relationship between the skill premium and the relative supply of skills. A negative relationship may be obtained moving on the lobbying axis. Note that the rise in the relative lobbying strength of the intermediate-goods firms in the H-sector may be the only reason the quantitative implications of the model indicate a positive relationship between the skill premium and the supply of skills. In fact, although the model predicts that high substitutability and scale effects combined may yield a positive relationship between the skill premium and the skill supply, when taking the model to data, one sees that this is not reasonable to occur.

In fact, one would need a higher elasticity of substitution and higher scale effects than the empirically reasonable values for a developed economy such as the USA for that to occur. Even if we consider such values, the yielding skill premium would also be much higher than the empirically reasonable current values for the USA.

The consideration of lobbying reconciles theory and empirical evidence according to which some historical periods and countries experience a positive relationship but some others experience a negative relationship between the skill premium and the relative supply of skills. The shape of this relationship changes considerably for the case of complementarity between skilled, H, and unskilled, L. In that case – Figure 1d) – an increase in the relative lobbying strength of the intermediate-goods firms in the H-sector would decrease wage inequality, following the result in Proposition 2. With substitutability, Figures 1a), 1b), and 1c), the H-augmenting technological-knowledge change is H-biased, then as the relative lobbying strength of the intermediate-goods firms in the H-sector increases, so, too, do its profitability and demand for skills, fostering their wages and increasing the skill premium. On the contrary, with complementarity, Figure 1d), the H-augmenting technological-knowledge change is L-biased, then as the relative lobbying strength of the intermediate-goods firms in the H-sector increases, so, too, does its profitability, but the relative demand for unskilled also increases, fostering their wages and decreasing the skill premium. This is the reason why the relationship between the relative lobbying strength of the intermediate-goods firms in the H-sector and the skill premium is negative with complementarity and positive with substitutability.

When comparing to the no-lobbying case, the possibility of lobbying always

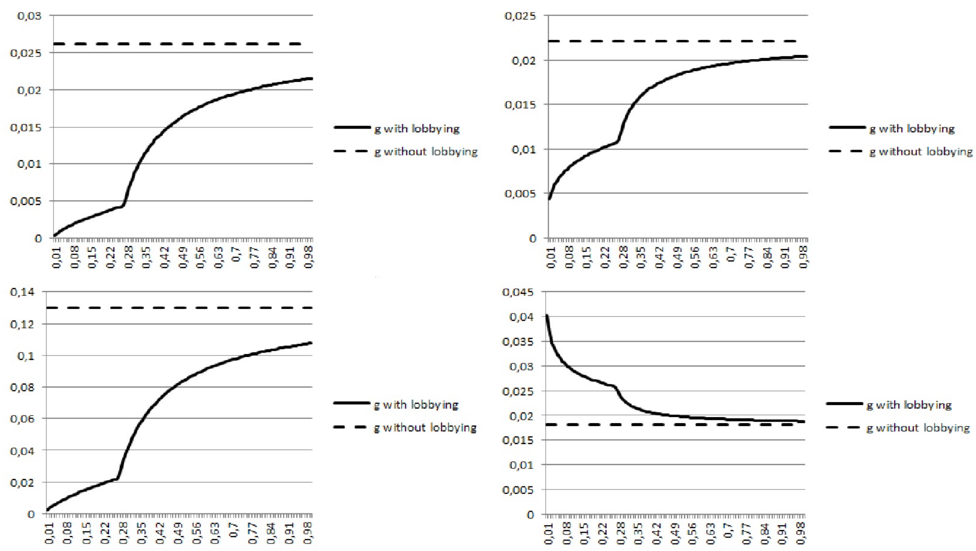

Figure 2: Economic growth rate, g , for different values of the lobbying effort, ϕ : a) Elasticity of Substitution, $\varepsilon = 2$, and no scale effects, $\delta = 1$; b) Elasticity of Substitution, $\varepsilon = 1.4$, and no scale effects, $\delta = 1$; c) Elasticity of Substitution, $\varepsilon = 2$, and intermediate scale effects, $\delta = 0.5$, adjusted $\sigma_H = 0.25$ and $\sigma_L = 0.05$ to yield reasonable growth rates).; d) Elasticity of Substitution, $\varepsilon = 0.8$, and no scale effects, $\delta = 1$.

decreases the skill premium when $\varepsilon > 1$, as the no-lobbying value is always higher than the highest value attained in Figures 1a), 1b), and 1c). However, if $\varepsilon < 1$, the no-lobbying case yields a lower skill premium than the lobbying cases, as the value for it without lobbying is always below the minimum obtained in Figure 1d). This is explained by the benefit L-firms gain with complementarity due to H-augmenting technological-knowledge change, which is enhanced by their own lobbying advantage. Thus, the lobbying cases skill premium tend to the no-lobbying case skill premium as the lobbying advantage of L gives place to an increasing lobbying advantage of H.

We now analyse the effect of lobbying on economic growth and Figures 2a), 2b), 2c), and 2d) present the results.

[Figure 2 goes about here]

The Figures show a high growth penalty introduced by the lobbying. In fact, depending on the lobbying effort and on most reasonable scenarios with null scale effects, the growth penalty from lobbying range from 0.2% to 0.5% in annual growth rate, i.e., with lobbying the USA economy grows 0.2% to 0.5%

less than it would without lobbying when there is substitutability, which is the most empirically appealing case. This is entirely due to the misallocation of resources that lobbying activities provoke in the economy. However, in the case of complementarity, there is a growth benefit of L-relative lobbying – see also the theoretical result in (43) –, which began at close to 2%, when L-firms have the lobbying advantage tending to 0% as the lobbying advantage of H-firms increases toward 1. This is explained by the benefit L-firms gain with complementarity due to H-augmenting technological-knowledge change, which is enhanced by their own lobbying advantage. As stated above, it is worth noting that the model accommodates both "growth miracles" (high-growth equilibrium), namely when $\varepsilon \to 0$, and "poverty traps" (low-growth equilibrium), namely when $\varepsilon \to +\infty$.

Additionally, both growth regimes introduced by lobbying are visible in the Figures. In particular there is: (i) a low-growth regime when the unskilled-sector firms have the lobbying advantage, where the economy growth is about 0% to 0.5% (a little more in Figure 2c), in which there are scale effects); (ii) a high-growth regime when the skilled-sector firms have the lobbying advantage, in which the growth rate rises until near 2% (in Figures 2a) and 3b), without scale effects) or until near 10% (in Figure 2c), with scale effects). As Proposition 1 also stresses, an increase in the relative lobbying strength of the intermediate-goods firms in the H-sector will raise the profitability of those firms that would bias technological-knowledge progress toward the goods produced in that sector and thus will benefit economic growth when substitutability implies that the H-sector benefits with their own technological-knowledge progress.

4 Concluding remarks

Lobbying is a growing activity in the most developed countries, namely in the USA and in the European Union. However, its effect on wage inequality has been overlooked in the literature. We fill this gap and introduce the possibility of lobbying in a generalized model of Directed Technical Change.

We discover that due to lobbying, the technological-knowledge bias becomes less directed toward skilled labour. Additionally, we show that an increase in the relative lobbying strength of the intermediate-goods firms in the H-sector would increase the skill premium under substitutability and decrease the skill premium under complementarity. This is because under substitutability the H-

augmenting technological-knowledge change is H-biased and then as the relative lobbying strength of the intermediate-goods firms in the H-sector increases, so, too, do its profitability and the demand for skills, fostering their wages and increasing the skill premium. On the contrary, under complementarity the H-augmenting technological change is L-biased and then as the relative lobbying strength of the intermediate-goods firms in the H-sector increases, so, too, does its profitability, but the relative demand for unskilled labour also increases, raising their wages and decreasing the skill premium.

There is a threshold under which the L-firms have lobbying advantage and above which the H-firms have lobbying advantage. The combination of L-firms lobbying advantage with complementarity favours unskilled labour regarding wage inequality, while substitutability and H-firms lobbying advantage favours the skill premium.

We have also uncovered effects of lobbying on economic growth and discovered that, under substitutability, the existence of lobbying implies a significant growth penalty but the relative lobbying power of H-firms tends to increase growth. On the contrary, under complementarity, the existence of lobbying implies a growth premium when the lobbying advantage is on the unskilled side, a premium that tends to vanish as the skilled lobbying advantage increases.

Moreover, we calibrate the model with empirically reasonable values for parameters and quantify the effects highlighted theoretically.

Lobbying exists and is increasing in modern societies. If the relative lobbying power of the skilled intensive firms increases, then growth increases along with wage inequality (the skill premium), inducing a trade-off between growth and inequality. This may call for redistributive policies in order to reduce inequality. However, if the relative lobbying power of the unskilled intensive firms increases, then growth and wage inequality decrease. This may reduce incentives to education and may call for public incentives, such as publicly provided education or subsidies.

Compliance with Ethical Standards

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Ethical approval. Both authors approve that their work complies with the ethical standards of the editorial to which the present paper was submitted.

Informed consent. Both authors of this paper agree with the submission of the paper to the Journal of the Spanish Economic Association (SERIEs). All of the data used in this paper were obtained from publicly available sources.

References

Acemoglu, D. (1998). "Why do new technologies complement skills? directed technical change and wage inequality." Quarterly Journal of Economics 113(4), 1055-1590.

Acemoglu, D. (2002). "Directed technical change." Review of Economic Studies 69(4), 781-809.

Acemoglu, D. (2003). "Cross-country Inequality Trends." Economic Journal 113(485), 121-149.

Acemoglu, D. (2008). Introduction to modern economic growth. Princeton University Press.

Acemoglu, D. and F. Zilibotti (2001). "Productivity differences." Quarterly Journal of Economics 116, 563-606.

Afonso, O. (2006). "Skill-biased technological knowledge without scale effects." Applied Economics 38(1), 13-21.

Afonso, O. (2008). "The impact of government on wage inequality without scale effects." Economic Modelling 25(2), 351-362.

Afonso, O. (2012). "Scale-independent North-South trade effects on the technological-knowledge bias and on wage inequality." Review of World Economics 148(1), 181-207.

Autor, D. (2014). "Skills, education, and the rise of earnings inequality among the other 99 percent." Science 344, 843-651.

Alesina, A., E. Spolaore, and R. Wacziarg (2005). "Trade, growth and the size of countries." Handbook of Economic Growth, in: Philippe Aghion & Steven Durlauf (ed.), Handbook of Economic Growth, edition 1, volume 1, chapter 23, pages 1499-1542 Elsevier.

Baier, S., M. Jerzmanowski, and R. Tamura (2012). "Directed technological change: a quantitative analysis." Mimeo.

Barro, R. and X. Sala-i-Martin (2004). Economic Growth. MIT Press (second edition), Cambridge, Massachusetts.

Bergoeing, R., N.V. Loayza, and F. Piguillem (2010). "Why are developing countries so slow in adopting new technologies." Policy Research Working Paper No. 5393, The World Bank.

Bound, J. and G. Johnson (1992). "Changes in the structure of wages in the 1980s: an evaluation of alternative explanations." American Economic Review 82(3), 371-392.

Chen, X., F. Favilukis, and S. Ludvigson (2013). "An estimation of economic models with recursive preferences." Quantitative Economics 4, 39-83.

Cole, H.L., L.E. Ohanian, A. Riascos, and J.A. Schmitz Jr (2005). "Latin America in the rearview mirror." Journal of Monetary Economics 52(1), 69-107.

Comin, D. and B. Hobjin (2009). "Lobbies and technology diffusion." Review of Economics and Statistics 91(2), 229-244.

Cothren, R. and R. Radhakrishnan (2017). "Productivity growth and welfare in a model of allocative inefficiency." Journal of Economics. https://doi.org/10.1007/s00712-017-0575-z.

Craig, J. and D. Madland (2014). "How campaign contributions and lobbying can lead to inefficient economic policy." Center for American Progress, accessed in July 2016: https://www.americanprogress.org/issues/economy/report/2014/05/02/88917/howcampaign-contributions-and-lobbying-can-lead-to-inefficient-economic-policy/

Crinò, R. (2005). "Wages, skills and integration in Poland, Hungary and Czech Republic: an industry-level analysis." Transition Studies Review 12(3), 432-459.

Duffy, J. and C. Papagiourgiou (2000). "A cross-country empirical investigation of the aggregate production function specification." Journal of Economic Growth 5, 87-120.

Esteban, J. and D. Ray (2006). "Inequality, lobbying, and resource allocation." American Economic Review 96(1), 257-279.

Gancia, G. and A. Bonfiglioli (2008). "North-South trade and directed technical change." Journal of International Economics 76(2), 276-295.

Goldin, C. and L. Katz (2007). "The race between education and technology: the evolution of the US wage differencial 1890 to 2005." NBER Working Paper 12984.

Grossmann, V. and M. Steger (2008). "Anti-competitive conduct, in-house R&D, and growth." European Economic Review 52, 987-1008.

Hall, R.E. and C.I. Jones (1999). "Why do some countries produce so much more output per worker than others." Quarterly Journal of Economics 114(1), 83-116.

Jones, C.I. (1995a). "R&D based models of economic growth." Journal of Political Economy 103(4), 759-784.

Jones, C.I (1995b). "Time series tests of endogenous growth models." Quarterly Journal of Economics 110(2), 495-525.

Jones, C.I. (2011). "Misallocation, economic growth, and input-output economics." NBER Working Paper No. 16742.

Juhn, C., K. Murphy, and B. Pierce (1993). "Wage inequality and the rise in returns to skill." Journal of Political Economy 101(3), 410-442.

Katz, L. and K. Murphy (1992). "Changes in relative wages, 1963-1987: supply and demand factors." Quarterly Journal of Economics 107(1), 35-78.

Le Breton M., P. Sudhlter, and V. Zaporozhets (2012). "Sequential legislative lobbying." Social Choice and Welfare 39(2), 491-520.

Olson, M. (1982). "The rise and decline of nations." New Haven: Yale University Press.

Olson, M. (1996). "Big bills left on the sidewalk: why some nations are rich, and others poor." Journal of Economic Perspectives 10(2), 3-24.

Mathur, I., M. Singh, F. Thompson, and A. Nejadmalayeri (2013). "Corporate governance and lobbying strategies." Journal of Business Research 66, 547-553.

Parente, S.L. and E.C. Prescott (1994). "Barriers to technology adoption and development." Journal of Political Economy 102(2), 298–321.

Parente, S.L. and E.C. Prescott (1999). "Monopoly rights: a barrier to riches." American Economic Review 89(5), 1216-1233.

Peretto, P. (1998). "Technological change and population growth." Journal of Economic Growth 3, 283-311.

Robertson, R. (2004). "Relative prices and wage inequality: evidence from Mexico." Journal of International Economics 64, 387-409.

Zhu, S.C. and D. Trefler (2005). "Trade and inequality in developing countries: a general equilibrium analysis." Journal of International Economics 65, 21-48.