Monetary Policy in an Endogenous Growth Model with R and Human Capital Accumulation

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Abstract

Despite some recent evidence according to which different inflation rates have effects on long-run growth, endogenous growth theory had advanced little on explaining the mechanics of monetary influence on economic growth. We follow the increasing interest in the issue offering a new explanation for the influence of monetary policy on growth in both long and short run: the cash requirements for households expenditures in education. Quantitatively, the model replicates both the small influence of monetary policy on growth while also highlighting the effects it can have on welfare and allocations of resources throughout different sectors in the economy.

Keywords: endogenous economic growth, inflation, interest rate, monetary policy, cash-in-advance (CIA).

JEL Classification: O30, O40, E13, E17, E61.

1 Introduction

With some remarkable exceptions that pointed out for very small effects of monetary variables on growth (see, for example, Jones and Manuelli, 1995; Chari et al., 1995), most endogenous growth theory had ignored the mechanics under which monetary policy can influence endogenous growth. The reason is because these seminal papers have identified very small effects or pointed out to the *superneutrality* of monetary policy.

More recent empirical results and models' steady-state effects were reported to be non-linear such that only above a given inflation threshold, the effects of inflation on economic growth are quantitatively relevant (see Bick, 2010 and Arawatari et al., 2018). This means that we should observe very small effects of inflation on growth for low inflation rates and somewhat higher effects for high inflation rates. López-Villavicencio and Mignon (2011) presents empirical evidence according to which there exists a threshold beyond which inflation exerts a negative effect on growth, and below which it is growth enhancing for advanced countries.

As is recognized by Klump and LaGrandville (2011) and Klump and Jurkat (2018), empirical research has pointed out for negative effects of inflation on growth and simultaneously economic growth theory has been dominated by the superneutrality result and that this is one of the major disappoints of the current economics science. On the one hand, most central banks are constitutionally committed to price stability. On the other hand, price instability seems to have important short-run welfare effects. In fact, Fischer (1979) and Cohen (1985) were able to show analytically that inflation can hasten convergence toward the steady state. Thus explaining the inflation-growth nexus is paramount. With this paper we wish to offer

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alternative explanations of the inflation-growth nexus and to study the (different) effects inflation has in different sources of economic growth and in transitional dynamics.

The papers most related to ours are Chu and Cozzi (2014), Arawatari et al. (2018), Klump and Jurkat (2018) and Gil and Iglésias (2019). The first highlights the effects of CIA constraint on R&D performance and economic growth, focusing of the distortionary effects of the CIA constraint on the optimality of R&D investments. The second stresses the non-linear influence of inflation on growth in a R&D based endogenous growth model introducing a specific skills distribution to obtain a relationship close to that of the empirically relevant one. Both articles included both fully and semi-endogenous R&D-based endogenous growth models. However, they abstain from analyzing transitional dynamics and restricts the analysis to the steady-state. Klump and Jurkat (2018) analyze the effect of elasticity of substitution on the convergence speed in both the steady state and the transitional dynamics. Finally, Gil and Iglésias (2019) explore complementarities between R&D and physical capital to study the effects of monetary policy on economic growth rate, real interest rate, physical investment rate, capital-to-labour ratio, R&D intensity, and velocity of money. Interestingly the relationship between inflation and economic growth in Gil and Iglésias (2019) is a negative (convex) one – qualitatively similar to the one we obtain in our baseline model – contradicting the main finding in Arawatari et al. (2018) which points out to a negative (concave) one with a specific form. However, quantitatively the effects of inflation on growth in our baseline framework is much closer to the small empirically validated effect. In an extension to our model, we feature a theoretical concave and nonlinear relationship between inflation and economic growth that is conforming the one in Arawatari et al. (2018).

We differ from all in two fundamental features.

First, we incorporate monetary policy in an endogenous growth model with several sources of long-run growth (human capital accumulation, and R&D through increasing varieties and improving qualities). The crucial difference is that the introduction of human capital accumulation changes the structure of the sources of growth, making endogenous growth rates relying on the human capital accumulation features as well as eliminating scale effects. To our knowledge this is the first paper to analyze the effects of monetary policy in the allocation of resources between production, education, and two different R&D sectors through the transitional dynamics of an endogenous growth model simultaneously considering R&D and human capital accumulation. We do that both at the steady-state and during the transitional dynamics within a common Cobb-Douglas and homogeneous skills distribution framework.

Second, we offer alternative explanations for small effects of inflation on economic growth following monetary policies on the transitional dynamics and also for non-linear effects in the long run. Furthermore, we calculate welfare effects of those policies. While the simple costs associated with a cash-in-advance constraint may account for small growth effects (which are potentially different for different growth sources), a non-linear private cost for education is needed to account for the empirically plausible non-linear inflation-growth nexus in the long run.

The paper is organized as follows. After this Introduction, in Section 2, we present an endogenous monetary-growth model with human capital accumulation and horizontal and vertical R&D. In Section 3 we present the steady-state analysis and the most important theoretical results. In Section 4 we derive the economic dynamic system. In Section 5, we simulate the economy evolution following a monetary policy shock on the nominal interest rate and study the influence of several monetary features on the growth rate and welfare (which takes in account transitional dynamics) and we offer results of a number of extensions of the baseline model. In Section 6 we conclude.

2 The Model

In this section we devise the model. The model includes both human capital accumulation R&D through both increasing varieties and quality-ladders, following Arnold (1998), Funke and Strulik (2000) and Strulik (2005). Additionally, we introduce the possibility of monetary policy and the study of its effects on growth in both the steady state and during the transitional dynamics.

2.1 Households

The representative agent maximizes intertemporal utility in order to consumption C

$$U = \int_0^\infty \frac{C_t^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt, \tag{1}$$

where $\rho > 0$ denotes the time preference rate, and $1/\theta > 0$ is the elasticity of intertemporal substitution. Agents take the aggregate rate of innovation as exogenous. They also earns returns, r, per unit of aggregate wealth, A_t, and retain real money holdings, $m_t = \frac{M_t}{P_t}$, with M_t being nominal money supply and P_t the price level in the economy. They invest in their own education, which stock is denoted by H_t, to which he devotes human resources H_H. The education effort is also subject to a opportunity cost due to being out of the labor market (which is the common assumption). It is also subject to a cost of education p,[1] which leads to a budget constraint[2]

$$\dot{A} + \dot{m} = rA_t + w_t(H_t - (1+p)H_{H_t}) - \mu m_t - C_t + T_t \tag{2}$$

where w is the real wage from labor, μ is inflation rate, C_t is is consumption, and T_t represents lump-sum transfers (or taxes, if it is negative) to households. Human capital is accumulated according to

$$\dot{H} = \xi H_{H_t}, \qquad \xi > 0. \tag{3}$$

Besides human capital accumulation itself, human capital detained by the households is used in the different market sectors in the economy: the intermediate goods production (H_{x_t}); the increasing variety R&D sector (H_{n_t}) and the quality ladder R&D sector (H_{Q_t}), such that:

$$H_t = H_{H_t} + H_{x_t} + H_{n_t} + H_{q_t}, \tag{4}$$

which in each moment in time, is supplied inelastically. Households (and firms) are also subject to cash-in-advance (CIA) constraint ensuring that some of the costs they face must be paid in money, thus:

$$m_t = \Theta_c C_t + \Theta_p p w_t H_{H_t} + \Theta_x w_t H_{x_t} + \Theta_n w_t H_{n_t} + \Theta_Q w_t H_{Q_t}. \tag{5}$$

[1]The cost of education (or the *fee*) is a very realistic feature in most developed countries that has been overlooked by the endogenous growth literature. It is a proportion of the wage the agent (or his family) earns in the labor market. This can be thought as fee dependent on the family labor income (as happens e.g. in the USA through the expected family contribution since the 1965 Higher Education Act) or a wage-contingent payment of a loan the agent takes out to earn a college degree. As showed in Brossard et al. (2015), a non-negligible percentage of costs of education are supported by the households, even for primary schooling and also in most developed countries. For example, in Spain and the United Kingdom the percentage of the households supporting costs with primary education is around 10% while in South Korea it reaches almost 20%. This cost proportion rises to 25% for secondary education in Japan and South Korea. In Spain, New Zealand, Italy and Lithuania families support 20% to 30% of tertiary education (colleges and universities) while this value rises to nearly 75% in the USA, United Kingdom, and Japan.

[2]All variable x variation is denoted by $\dot{x}$. Also, let g_x denote x's growth rate, $g_x = \dot{x}/x$.

The first-order conditions for an interior solution yield[3]

$$g_C = \frac{r-\rho}{\theta}, \tag{6}$$

$$g_w = r - \frac{\xi}{1+p(1+i\Theta_p)}. \tag{7}$$

The first of these equations is the standard Ramsey rule. The second indicates that investment in human capital is ensured when the overall return on human capital $g_w + \frac{\xi}{1+p(1+i\Theta_p)}$ is equal to the return on financial assets r. It is worth noting that the non-opportunity cost introduced in this paper is relevant in face of the monetary policy that targets nominal interest rate. Otherwise, the non-opportunity cost p, although empirically relevant would be equivalent to a discount of the productivity of learning in the wages and economic growth rate.

2.2 Monetary Authority

The monetary authority controls the nominal interest rate i, which is kept constant over time ($i_t = i \geq 0$). This also means that the monetary authority is in equilibrium adjusting inflation such that nominal interest rate is constant. For the private agents inflation is taken as given. The monetary authority rebates seigniorage revenue to households through lump-sum transfers. Then, $T = m$ holds.

2.3 Firms and Markets

2.3.1 Intermediate Goods

Intermediate goods are used to produce a final good Y that sells on a competitive market at a price of one. Technology is constant returns to scale with elasticity of substitution $\sigma > 1$. Intermediate goods are characterized by quality and quantity. After k innovations in quality research, a good $j = 1, ..., n$ is available at qualities q_{k_j}, with $k_j = 0.1, ...\kappa_j$ which is used in quantities x_{k_j}. Hence, aggregate output can be written as:

$$Y = [\int_0^n (\sum_{kj=0}^{kj} q_{kj} x_{kj})^{1-1/\sigma} dj]^{1/(1-1/\sigma)} \tag{8}$$

Profit maximization yields demand for intermediates:

$$x_{k_j} = \frac{q_{k_j}^{\sigma-1} p_{k_j}^{-\sigma} Y}{\int_0^n \left(\frac{q_{k_l}^{\sigma-1}}{p_{k_l}^{\sigma-1}}\right) dl}, \tag{9}$$

where p_{k_j} denotes the price of a unit of x_{k_j}. Each intermediate good is produced using human capital such that $x_{k_j} = H_{x_j}$. Intermediate goods are produced by monopolistically competitive firms which take wages w as given and maximize profits

$$\pi_{k_j} = p_{k_j} x_{k_j} - w H_{x_j} - i\Theta_x w H_{x_j}. \tag{10}$$

to which costs are composed by the usual real labor (human capital) costs and the costs associated with the CIA constraint. Substitution of (9) – after substituting (8) – in (10) this yields prices:

$$p_{k_j} = w \frac{\sigma(1+i\Theta_x)}{\sigma - 1} \tag{11}$$

Interestingly, this gives the result that money supply implying higher inflation and nominal interest

[3]The time subscript t is eliminated from now on for simplification purposes.

rate also influences the prices of intermediate goods. Inserting (11) into (9) yields supply of good j according to

$$x_{k_j} = \frac{\sigma - 1}{\sigma(1 + i\Theta_x)} \frac{q_{k_j}^{\sigma-1} Y}{wnQ}. \tag{12}$$

Therefore, instantaneous profits of the firm supplying good j are:

$$\pi_j = \frac{1}{\sigma} \frac{q_{k_j}^{\sigma-1} Y}{nQ}. \tag{13}$$

This means that intermediate firms profits are not affected by monetary policy.

2.3.2 Research through increasing varieties

It is worth noting that research occurs both due to the increasing number of technologies (increasing varieties) and raising their quality. First we will characterize the sector in which new technologies are developed.

New technologies appear in the economy as a results of the activity of an increasing-varieties sector such that:

$$\dot{n} = An^{\beta_1} Q^{\beta_2 - 1} H_n^{1-\chi}, \tag{14}$$

where n is the number of varieties in the economy, Q is the aggregate level of quality, H_n is the part of human capital that is allocated to the horizontal R&D sector, β_1 measures the typical spillover effect within the increasing-varieties R&D sector, β_2 is the cross-sector spillover from the quality sector to the varieties sector and χ measures the duplication effect.

Firms in the increasing-varieties sector maximize their expected value (the number of varieties produced times its value) minus research cost:

$$\pi_n = \dot{n} V_n - wH_n - i\Theta_n wH_n. \tag{15}$$

The free entry condition is thus: $V_n \dot{n} = w(1 + i\Theta_n) H_n$ implying that:

$$\frac{\dot{V}_n}{V_n} = \frac{\dot{w}}{w} - \beta_1 \frac{\dot{n}}{n} - (\beta_2 - 1)\frac{\dot{Q}}{Q} + \chi \frac{\dot{H}_n}{H_n}. \tag{16}$$

This means that the value of an innovation times the number of innovations must be equal to the cost of producing those innovations.

The non-arbitrage condition for the increasing-varieties firm states that: $r = \pi_{0j}/V_n + \frac{\dot{V}_n}{V_n} - \varsigma \frac{V_{0_j}}{V_n}$, meaning that the return from investing in assets (rV_n) plus the variety innovators' expected loss through the first quality improvement (ς) times the value of that innovation (V_{0_i}) must be equal to the instant profits from selling the respective intermediate good (produced with the state-of-the-art technology) (π_{0_j}) plus the valorization of the patent ($\dot{V}_n$). Note that ςV_{0_j} denotes the variety innovators' expected loss through the first quality improvement. Then $V_{0_j} = V_{k_j} \frac{Q}{q_{k_j}^{\sigma-1}}$.

Inserting the free-entry condition yield by (15) into the no-arbitrage condition provides:

$$r = \frac{1}{\sigma} \frac{An^{\beta_1 - 1} Q^{\beta_2 - 1} H_n^{-\chi} Y}{w(1 + i\Theta_n)} + \frac{\dot{w}}{w} - \beta_1 \frac{\dot{n}}{n} - (\beta_2 - 1)\frac{\dot{Q}}{Q} + \chi \frac{\dot{H}_n}{H_n} - \varsigma Q \frac{V_{k_j}}{V_n} \frac{1}{q_{k_j}^{\sigma-1}}. \tag{17}$$

2.3.3 Research through quality-ladders

As said before, there is also a quality-ladders sector. At each point in time an improvement from quality level k to $k+1$ occurs with probability:

$$\varsigma_{k_j} = \frac{An^{\alpha_1}Q^{\alpha_2}H_Q^{-\chi}}{q_{k_j}^{\sigma-1}}H_Q \tag{18}$$

Thus, quality increases follow the accumulation function

$$\dot{Q} = (\gamma^{\sigma-1} - 1)An^{\alpha_1-1}Q^{\alpha_2}H_Q^{1-\chi}, \tag{19}$$

where α_1 measures the cross-sector spillovers from the varieties sector into the quality-ladder sector, and α_2 is the spillover within the quality ladders sector. The parameter γ measures the increase in quality within each quality sector and thus the term $\gamma^{\sigma-1} - 1$ measures the creative destruction effect. The allocation of human capital to the vertical R&D sector is H_Q.

Firms in the quality-ladder sector maximize expected value minus research cost:

$$\pi_Q = \varsigma_{k_j}V_{k_j} - wH_{Q_j} - i\Theta_Q wH_Q, \tag{20}$$

where ς_{k_j} is the probability of success of a vertical innovation and w is the wage of human capital. The free-entry condition is: $\varsigma_{k_j}V_{k_j} = w(1 + i\Theta_Q)H_{Q_j}$ which implies

$$\frac{\dot{V}_{k_j}}{V_{k_j}} = \frac{\dot{w}}{w} - \alpha_1\frac{\dot{n}}{n} - \alpha_2\frac{\dot{Q}}{Q} + \chi\frac{\dot{H}_Q}{H_Q}. \tag{21}$$

This means that the value of the innovation times the number of quality improvements made on the available technologies (varieties) must be equal to the cost of producing these innovations.

The non-arbitrage condition for the quality-ladders firm states that: $r = \pi_{k_j}/V_{k_j} + \frac{\dot{V}_{k_j}}{V_{k_j}} - \varsigma_{k_j}$, meaning that the return from investing in assets (rV_{k_j}) must be equal to the instant profits from selling the respective state-of-the-art quality intermediate good (π_{k_j}) plus the increase in the value of the patent ($\dot{V}_{k_j}$) minus the probability that a competitor succeeds in research and drives the incumbent out of business (ς_{k_j}). Inserting (13), using (18), and free-entry condition yield by (20) into the no-arbitrage condition provides:

$$r = \frac{1}{\sigma}\frac{An^{\alpha_1-1}Q^{\alpha_2-1}H_Q^{-\chi}Y}{w(1+i\Theta_Q)} + \frac{\dot{w}}{w} - \alpha_1\frac{\dot{n}}{n} - \alpha_2\frac{\dot{Q}}{Q} + \chi\frac{\dot{H}_Q}{H_Q} - \varsigma_{k_j}, \tag{22}$$

2.4 Aggregate Innovation, money and labor demand

If an innovation occurs in a sector j, quality grows at rate $(\gamma^{(k_j+1)(\sigma-1)} - \gamma^{k_j(\sigma-1)})/\gamma^{k_j(\sigma-1)} = \gamma^{\sigma-1} - 1$. In any sector at any time, an innovation occurs with probability m per unit of time Hence, expected growth of the quality index is $\frac{\dot{Q}}{Q} = \varsigma(\gamma^{\sigma-1} - 1)$. Demand for research in quality research is given by transforming (18) in

$$H_Q = \int_0^n H_{Q_j}dj = \frac{\varsigma\int_0^n q_{k_j}^{(\sigma-1)}dj}{n^{\alpha_1}Q^{\alpha_2}H_Q} = \frac{\varsigma nQ}{n^{\alpha_1}Q^{\alpha_2}H_Q}. \tag{23}$$

Integrating (12), we obtain the demand for human capital in the intermediate good sector H_x:

$$H_x = \frac{\sigma-1}{\sigma(1+i\Theta_x)}\frac{Y}{w}, \tag{24}$$

which implies that $\frac{\dot{H}_x}{H_x} = g_Y - g_w$. By (5), we reach:

$$1 = \Theta_c\frac{C_t}{m_t} + \Theta_p p\frac{w_tH_{H_t}}{m_t} + \Theta_x\frac{w_tH_{x_t}}{m_t} + \Theta_n\frac{w_tH_{n_t}}{m_t} + \Theta_Q\frac{w_tH_{Q_t}}{m_t}. \tag{25}$$

The final good output solving (8) and taking into account that $Q = \int_0^n q_j^{\sigma-1} dj$, can be written as follows:

$$Y = (nQ)^{\frac{1}{\sigma-1}} H_x. \tag{26}$$

Combining growth rates resulting from (24) and (26) the growth rate of wages is:

$$g_w = \frac{1}{\sigma-1}(g_n + g_Q). \tag{27}$$

This equation tells us that the growth rate of real wages is proportional by the technological growth rate in the economy.

3 Steady-State Analysis

First, we obtain a modified version of the no-arbitrage conditions after consideration of the free-entry conditions in R&D, i.e., modified versions of equations (17) and (22), respectively. Using (7), (14), (24), and the fact that the expected growth of the quality index is $\frac{\dot{Q}}{Q} = \varsigma(\gamma^{\sigma-1} - 1)$, we obtain the first of the following equations. Then, using (7), (19), (24) we obtain the second of the following equations.[4]

$$\frac{\xi}{1+p(1+i\Theta_p)} = \left(\frac{H_x}{H_Q}\frac{1+i\Theta_x}{(\sigma-1)(1+i\Theta_Q)} - 1\right)\frac{\dot{Q}}{Q}\frac{1}{\gamma^{\sigma-1}-1} - \alpha_1\frac{\dot{n}}{n} - \alpha_2\frac{\dot{Q}}{Q} + \chi\frac{\dot{H}_Q}{H_Q} \tag{28}$$

$$\frac{\xi}{1+p(1+i\Theta_p)} = \left(\frac{1+i\Theta_x}{(\sigma-1)(1+i\Theta_n)}\frac{H_x}{H_n} - \frac{1+i\Theta_Q}{(1+i\Theta_n)}\frac{H_Q}{H_n}\right)\frac{\dot{n}}{n} - \beta_1\frac{\dot{n}}{n} - (\beta_2 - 1)\frac{\dot{Q}}{Q} + \chi\frac{\dot{H}_n}{H_n}. \tag{29}$$

Second, as usual in growth theory, we define a steady state in which both the growth rates and allocation of human capital to the different sectors in the economy are constant. Using (7) we obtain a value for r that can be substituted in (6). Using (27) we obtain the economic growth rate at the steady state as

$$g_Y = \frac{1}{\theta}\left(\frac{\xi}{1+p(1+i\Theta_p)} + \frac{1}{\sigma-1}(g_n + g_Q)\right). \tag{30}$$

Furthermore, using R&D technologies (14) and (19) and assuring that growth rates are constant yield that:

$$g_n = \frac{(\alpha_2 - \beta_2)(1-\chi)}{(1-\alpha_1)(1-\beta_2) - (1-\alpha_2)(1-\beta_1)} g_H, \tag{31}$$

and

$$g_Q = \frac{(\beta_1 - \alpha_1)(1-\chi)}{(1-\alpha_1)(1-\beta_2) - (1-\alpha_2)(1-\beta_1)} g_H. \tag{32}$$

For positive growth, one of the following pairs of conditions has to be fulfilled: $\alpha_2 > \beta_2$ and $\beta_1 > \alpha_1$ or $\alpha_2 < \beta_2$ and $\beta_1 < \alpha_1$.

Inserting (31) and (32) into (30) and using the growth rate version of (26) provide steady-state growth of human capital as solely determined by the models parameters of technology and preference:

$$g_H = \frac{(\frac{\xi}{1+p(1+i\Theta_p)} - \rho)/(\sigma-1)}{\frac{(\theta-1)(1-\chi)(\beta_1-\alpha_1+\alpha_2-\beta_2)}{(1-\alpha_1)(1-\beta_2)-(1-\alpha_2)(1-\beta_1)} + \theta}. \tag{33}$$

Inserting (33) in (30) after using (31) and (32) – taking into account (25) such that money holdings are constant at the steady state – yields steady-state growth of consumption, money holdings or output *per capita* as determined solely by the model's parameters of technology and preference:

[4] For the second parcel in the term in parentheses in equation (29) we need to use free-entry conditions $V_n\dot{n} = w(1+i\Theta_n)H_n$, $\varsigma_{k_j}V_{k_j} = w(1+i\Theta_Q)H_Q$ and the following already mentioned definitions $V_{0_j} = V_{k_j}\frac{Q}{q_{k_j}^{\sigma-1}}$ and $Q = \int_0^n q_j^{\sigma-1} dj$.

$$g_c = g_m = g_Y = \frac{\frac{\xi}{1+p(1+i\Theta_p)} - \rho}{\theta - 1}\left(1 - \frac{1}{\frac{(\theta-1)(1-\chi)(\beta_1-\alpha_1+\alpha_2-\beta_2)}{(1-\alpha_1)(1-\beta_2)-(1-\alpha_2)(1-\beta_1)} + \theta}\right). \tag{34}$$

Regarding the relationship between inflation and growth, we arrive to the following proposition.

Proposition 1. *The effect of inflation in the steady-state growth rate is negative, monotonous, and nonlinear.*

Proof. From (30), obtain $\frac{\partial g_Y}{\partial \mu}$ and use the Fisher equation. This yields: $-\left(\Upsilon/\left(\theta-1\right)\right)\frac{\xi p \Theta_p}{(1+p(1+i\Theta_p))^2} < 0$, where $\Upsilon = 1 - \frac{1}{\frac{(\theta-1)(1-\chi)(\beta_1-\alpha_1+\alpha_2-\beta_2)}{(1-\alpha_1)(1-\beta_2)-(1-\alpha_2)(1-\beta_1)}+\theta}$. □

The proposition makes it clear that the negative effect of inflation on growth is obtained due to a non-opportunity cost to education p – a fee. It is also interesting to note that the growth rate is always decreasing in the inflation rate. The negative effect of inflation on growth is also higher the higher the productivity of education (ξ). However, both the fee and the cash-in-advance parameter Θ_p have a non-linear relationship with the effect of inflation on growth. Exact quantitative effects are highlighted through calibration.

3.1 Comparative Steady-State Analysis

Following Strulik (2005) and Sequeira et al. (2013), we consider $\beta_1 = 0.25$, $\beta_2 = 0.25$, $\alpha_1 = 0.4$, $\alpha_2 = 0.1$, $\chi = 0.5$, $\sigma = 6.00$, $\xi = 0.0675$, $\rho = 0.02$, $\theta = 2$, $\gamma = 1.05$. As in Arawatari et al. (2018), we consider a baseline situation in which all expenditures are made in cash such that $\Theta_p = 1$ and we consider a fee that is 25% of the wage, thus $p = 0.25$. Figure 3 shows the relationship between economic growth and inflation rate in the steady state for a series of inflation ranging from 1% to 70%. Despite the theoretical result in the Proposition 1 that points to a non-linear (convex) relationship, Figure 1 highlights a quasi-linear (negative) relationship between inflation rate and economic growth. Note also the very small effect of rising inflation in the long run.[5] An increase in inflation between 1% and 60% implies a decrease of just 0.1 percentage point in the economic growth rate.

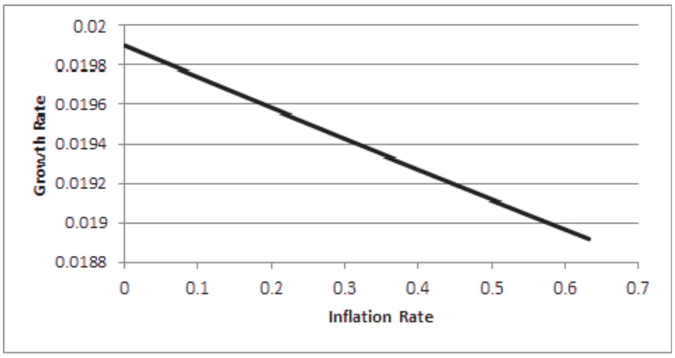


Figure 1: Relationship between steady-state growth rate and inflation rate

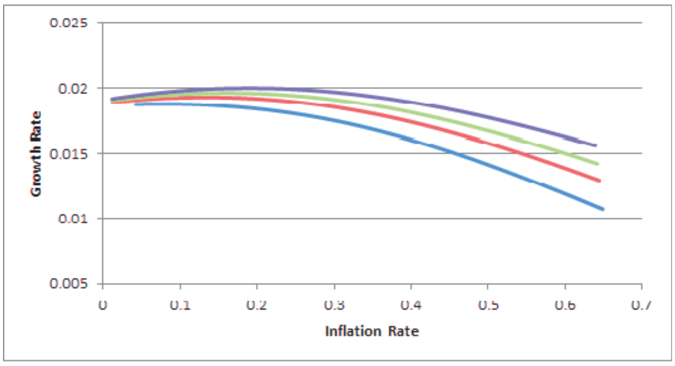
As the center of the nonneutrality result on long-run growth is the non-opportunity cost of education (*fee*), we propose that a non-linear structure for such a cost may be the explanation for the non-linearity that resembles the small effects for low inflation levels and high effects for quite high levels of the inflation rate. Suppose that the fee has a relationship with the inflation rate. Also, the fee $\hat{p}$ has a base value and a

[5]For example, Gil and Iglésias (2019) obtain also a convex negative effect of inflation on economic growth but with much higher quantitative effects (see their Figure 1). In their setup, an increase in inflation from 0% to 10% decreases growth in almost 4% while in our setup it only decreases in 0.02% which is much closer to empirical evidence.

variable value that is related to a given threshold for inflation. Intuitively, this means that the fee highly increases greatly in times of very high inflation.[6] Assume the following fee function:

$$\hat{p} = p + (i - \bar{i})^2 \tag{35}$$

If we set $\bar{i} = 0.25$ we can plot the long-run relationship between inflation and economic growth in Figure 2. This figure highlights three important facts. First, it allows for a slightly positive effects of inflation on growth for very low levels of inflation, a result that resembles the one pointed out by Fischer (1979) and Cohen (1985) but in our case for the steady-state. Second it also replicates the stronger (negative) effects of rising inflation on steady-state economic growth also highlighted by Arawatari et al. (2018). Third, the concavity of the relationship crucially depends on the money demand for education, parameter Θ_p in the cash-in-advance constraint. It is also worth noting that the relationship that best replicates the empirical relationship that has been reported in empirical studies (see also Arawatari et al., 2018) is the one that assumes the highest money requirement for schooling expenditures (blue line, with Θ_p=1).



Notes: blue line is for $\Theta_p = 1$, red line is for $\Theta_p = 0.75$, green line is for $\Theta_p = 0.25$, and purple line is for $\Theta_p = 0$.

Figure 2: Relationship between steady-state growth rate and inflation rate.

4 Stationary Variables and Transitional Dynamics

We will solve the dynamics of the model taking into account the following five stationary variables:

$$\upsilon_Q = \frac{Q}{H^{\frac{\beta_1-\alpha_1}{D}(1-\chi)}}; \tag{36}$$

$$\upsilon_n = \frac{n}{H^{\frac{\alpha_2-\beta_2}{D}(1-\chi)}}; \tag{37}$$

$$u_X = \frac{H_X}{H}, u_Q = \frac{H_Q}{H}, u_n = \frac{H_n}{H}. \tag{38}$$

where $D = (1-\alpha_1)(1-\beta_2) - (1-\alpha_2)(1-\beta_1)$. The first two are state variables and the last three are control variables. Based on these variables, we will derive a system of five differential equations.

By log-differentiation of (36), using the equations for the growth rate of qualities and the growth rate of human capital, given above, we obtain:

$$\frac{\dot{\upsilon_Q}}{\upsilon_Q} = (\gamma^{\sigma-1} - 1)A\upsilon_n^{\alpha_1-1}\upsilon_Q^{\alpha_2-1}u_Q^{1-\chi} - \frac{\beta_1-\alpha_1}{D}(1-\chi)\left[\xi\left(1-u_X-u_Q-u_n\right)\right] \tag{39}$$

[6]This is a reasonable assumption, as in times of high inflation the schools need to account for inflation to establish their fees.

By log-differentiating (37), using the equations for the growth rate of varieties and the growth rate of human capital given above we obtain:

$$\frac{\dot{\upsilon}_n}{\upsilon_n} = A\upsilon_n^{\beta_1-1}\upsilon_Q^{\beta_2-1}u_n^{1-\chi} - \frac{\alpha_2-\beta_2}{D}(1-\chi)\left[\xi\left(1-u_X-u_Q-u_n\right)\right] \tag{40}$$

By the log-differentiation of u_X, u_Q, and u_n, we obtain, respectively, the following dynamic equations. For the equation of u_X, we use the log-differentiated version of (26) and (27) and the above law of motion for human capital (see equations 3 and 4). For the equation for u_Q we use (28) and the law of motion for human capital. Finally, for the equation for u_n, we resort to (29) and to the law of motion for human capital.

$$\begin{aligned}\frac{\dot{u}_X}{u_X} &= \frac{1-\theta}{\theta}\frac{1}{\sigma-1}\left[A\upsilon_n^{\beta_1-1}\upsilon_Q^{\beta_2-1}u_n^{1-\chi}+(\gamma^{\sigma-1}-1)A\upsilon_n^{\alpha_1-1}\upsilon_Q^{\alpha_2-1}u_Q^{1-\chi}\right]\\ &\quad+\frac{1}{\theta}\left(\frac{\xi}{1+p(1+i\Theta_p)}-\rho\right)-\left[\xi\left(1-u_X-u_Q-u_n\right)\right]\end{aligned} \tag{41}$$

$$\begin{aligned}\frac{\dot{u}_Q}{u_Q} &= \frac{1}{\chi}\frac{\xi}{1+p(1+i\Theta_p)}-\frac{1}{\chi}\left(\frac{u_X}{u_Q}\frac{1+i\Theta_x}{(\sigma-1)(1+i\Theta_Q)}-1-\alpha_2(\gamma^{\sigma-1}-1)\right)A\upsilon_n^{\alpha_1-1}\upsilon_Q^{\alpha_2-1}u_Q^{1-\chi}+\\ &\quad+\frac{1}{\chi}\alpha_1A\upsilon_n^{\beta_1-1}\upsilon_Q^{\beta_2-1}u_n^{1-\chi}-\left[\xi\left(1-u_X-u_Q-u_n\right)\right]\end{aligned} \tag{42}$$

$$\begin{aligned}\frac{\dot{u}_n}{u_n} &= \frac{1}{\chi}\left[\frac{\xi}{1+p(1+i\Theta_p)}-\left(\frac{1+i\Theta_x}{(\sigma-1)(1+i\Theta_n)}\frac{u_X}{u_n}-\frac{1+i\Theta_Q}{(1+i\Theta_n)}\frac{u_Q}{u_n}-\beta_1\right)A\upsilon_n^{\beta_1-1}\upsilon_Q^{\beta_2-1}u_n^{1-\chi}\right.\\ &\quad\left.+(\beta_2-1)\left(\gamma^{\sigma-1}-1\right)A\upsilon_n^{\alpha_1-1}\upsilon_Q^{\alpha_2-1}u_Q^{1-\chi}\right]-\\ &\quad-\left[\xi\left(1-u_X-u_Q-u_n\right)\right]\end{aligned} \tag{43}$$

5 Transitional Dynamics

In this section we are not particularly interested in replicating the long-run non-linear relationship (which in fact becomes effective only after a relatively high value for the inflation rate). Alternatively we want to explore the allocation effects of a monetary policy during the transitional dynamics that follows the implementation of such a policy. After that, in a robustness analysis, we present results for growth and welfare effects of a given monetary policy within different models, corresponding to changes to the underlying assumption of the baseline model devised above.

5.1 Baseline analysis

We consider the following set of initial parameters: $\beta_1 = 0.25$, $\beta_2 = 0.25$, $\alpha_1 = 0.4$, $\alpha_2 = 0.1$, $\chi = 0.5$, $\sigma = 6.00$, $\xi = 0.0675$, $\rho = 0.02$, $\theta = 2$, $\gamma = 1.05$, $p = 0.25$.[7] As in Arawatari et al. (2018), we consider a baseline situation in which all expenditures are made in cash such that $\Theta_c = \Theta_p = \Theta_x = \Theta_n = \Theta_Q = 1$. Initial nominal interest rate set by the monetary authority is $i_0 = 0.10$ – corresponding to a high 10% inflation rate. This yields a steady-state growth rate of $g_Y = 1.75\%$. Then we introduce a monetary policy shock that fix the new nominal interest rate to $i_{new} = 0.06$. This will have a steady-state effect yielding a new $g_Y = 1.77\%$.

[7]This value is in line with evidence presented in Brossard et al. (2015: figure 32).

In an alternative exercise we consider lower values for parameters associated with the CIA constraint, assuming that some of the payments can be made directly with non-money assets. As argued in Arawatari et al. (2018) and in references therein (e.g. Schiffer and Weder, 2001 and Beck et al., 2005) smaller firms especially in countries with high inflation face more difficulties in doing business due to high inflation. Thus, higher values for parameters Θ should be associated with higher inflation. In a situation in which we want to simulate a more developed, higher growth economy, we may consider different values for different parameters $\Theta < 1$. In this case we assume that firms can make proportionally fewer payments in money than families and that R&D firms need higher cash flow for payments than intermediate goods firms (as also assumed in Chu and Cozzi, 2014). This means that $\Theta_c = \Theta_p = 0.9$; $\Theta_n = \Theta_Q = 0.7$ and $\Theta_x = 0.5$. Initial and final nominal interest rates set by the monetary authority are equal to the exercise described above. This yields an initial steady-state growth rate of $g_Y = 1.76\%$ and a final steady-state $g_Y = 1.78\%$. This exercise is done to evaluate the effects of having different cash-in-advance features. The immediate conclusion is that less cash holding requirements the higher the economic growth rate. However, a given change in the nominal interest rate will have almost the same effect in both scenarios: the one with complete cash requirements for expenditures and the one with only partial cash requirements for expenditures.

5.2 Growth, allocation, and welfare effects of monetary policy

Figure 1 illustrates the transitional dynamics for the baseline exercise described above for the main variables in the model. Variables υ_Q and υ_n are the state variables in the model such that they cannot jump after a monetary shock (see Figures 3a and 3b). Intuitively, these variables measure the weight of technological knowledge when compared to embodied knowledge (human capital) as they are ratios between each of the variety and quality indexes and human capital. They decrease following a monetary policy shock indicating that it the technological intensity in the economy decreases when compared to embodied knowledge intensity, resulting from the costs that R&D firms face due to inflation. Inflation decreases at once nearly 5% and 4.6% and then continues to slowly decrease toward its new steady-state value (see Figure 3c). Allocation effects are interesting as they are mainly due to a reallocation of resources to human capital accumulation from the production of the final good, which are induced by the policy (see Figures 3d and 3f). There are also transitional reallocations from R&D sectors to human capital accumulation but they are smaller especially due to the smaller dimension of these sectors in the economy (see Figures 3g and 3h). The monetary policy that decreases nominal interest rates and inflation is expansionary as it permanently increases the economic growth rate (see Figure 3e) both at the moment of the shock and through transition. However, as mentioned above and according to empirical evidence, the effects on economic growth are very small. This exercise uncovers, however, that the mechanics that yield a positive (although) small effect of inflation on growth relies on reallocation of resources toward human capital accumulation.

Figure 4 shows results of the alternative exercise with only partial cash holding requirements described above ($\Theta_c = \Theta_p = 0.9$; $\Theta_n = \Theta_Q = 0.7$ and $\Theta_x = 0.5$). When compared to the previous case (depicted at Figure 3) qualitative effects are similar and quantitative effects are also close to the previous ones. The only remarkable difference is a slightly higher reallocation from the R&D sectors to the human capital that can be seen in Figures 4g and 4h. This results are according to the ones in Chu and Cozzi (2014) and also empirical evidence according to which R&D activities' higher cash requirements implies a negative effect of inflation on R&D.

Figure 5 depicts the time path of consumption without and with the monetary policy shock. The figure highlights small changes that have an effect in welfare, amounting to an increase in 0.63% and

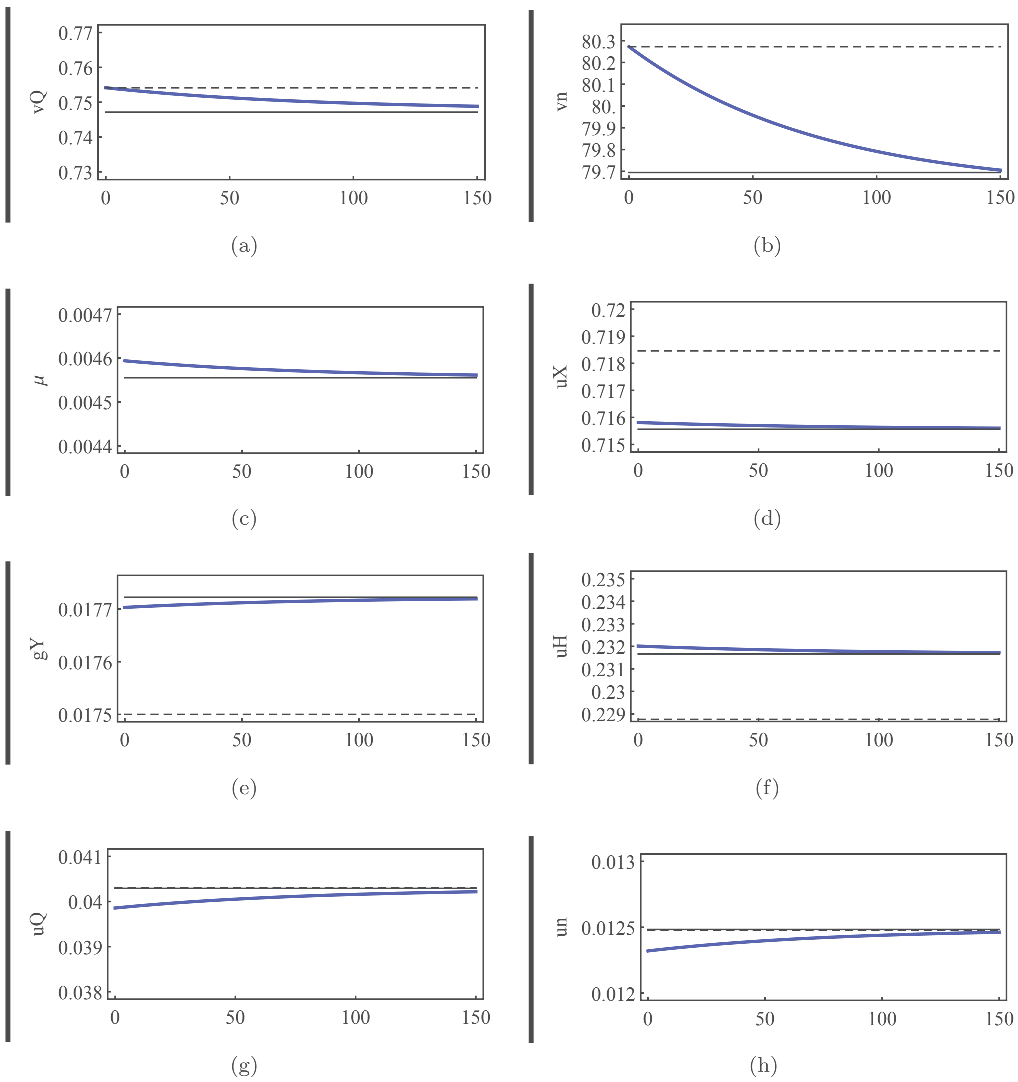


Figure 3: Effects of a shock on nominal interest rate i from 10% to 6%, with all expenses paid by cash. Note: Thin black line represents initial steady-state values. Dashed line represents final steady-state values. If they coincide, initial and final steady-states coincide, despite the transitional dynamics.

0.58%, respectively.

5.3 Robustness and effects on growth and welfare

In order to analyse the robustness of the main results to changes in functional forms of the model preferences and production functions, we perform a number of different changes and evaluate the effects on steady-state growth and allocations of a given monetary policy as well as on welfare (which takes into account the transitional dynamics after the policy change represented by 4 percentage points drop in the nominal interest rate (from 10% to 6%). Results are presented in Table 1. First, we want to evaluate the changes that may yield from the alternative education cost function proposed in Section 3.1. As becomes clear from the first line in the Table, we obtain a negative growth and welfare effect of decreasing inflation – resulting for a positive relationship between two rates, an effect opposite to what is obtained in the baseline analysis but consistent with the first part of the nonlinear relationship presented in Figure 2. In this case, there is a reallocation of human capital from the learning sector (and also from R&D sectors) to the final good sector which implies a decrease in the *per capita* output growth rate. This is due to the non-linear form of the education cost function (see again Figure 2). As expected by this nonlinear relationship, had we considered higher levels of the initial nominal interest rate (e.g. 30%), we would obtain

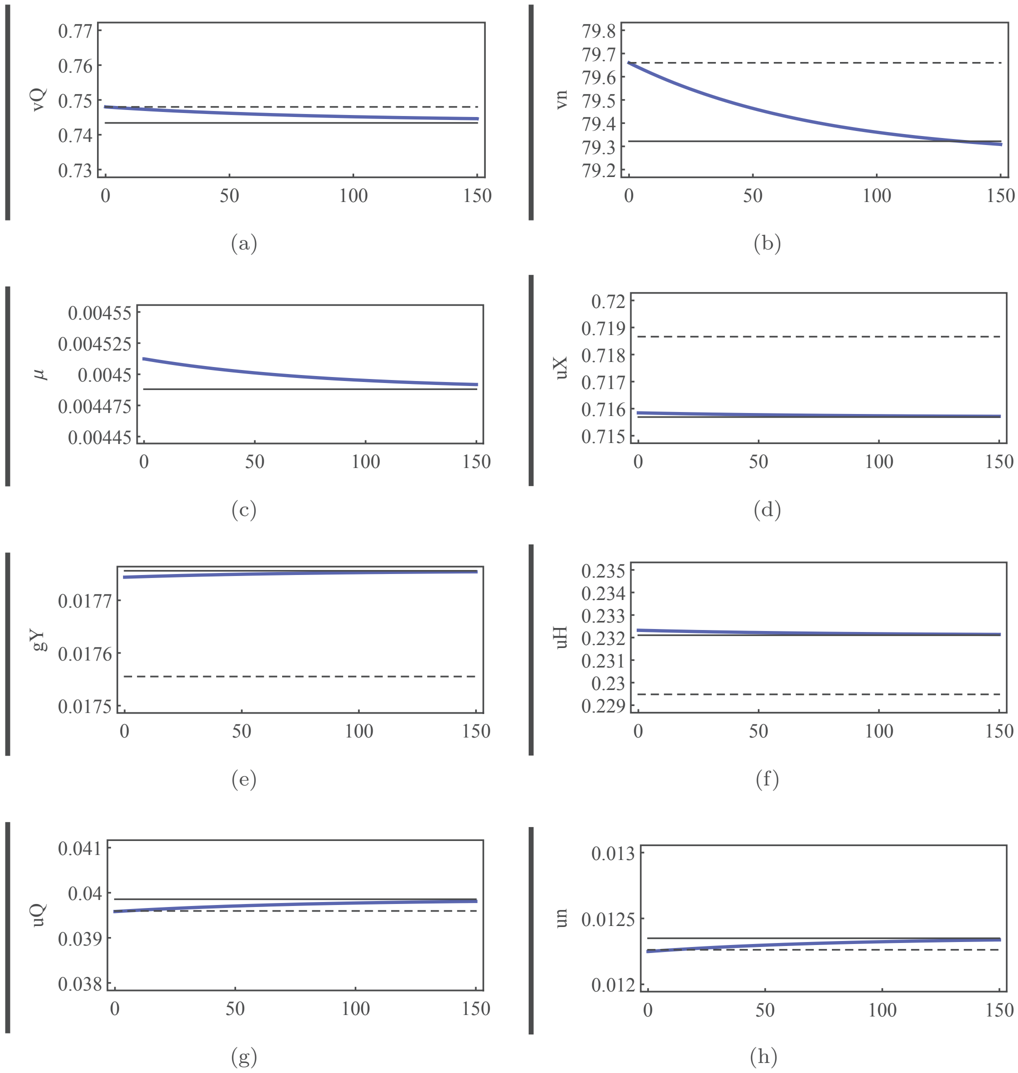
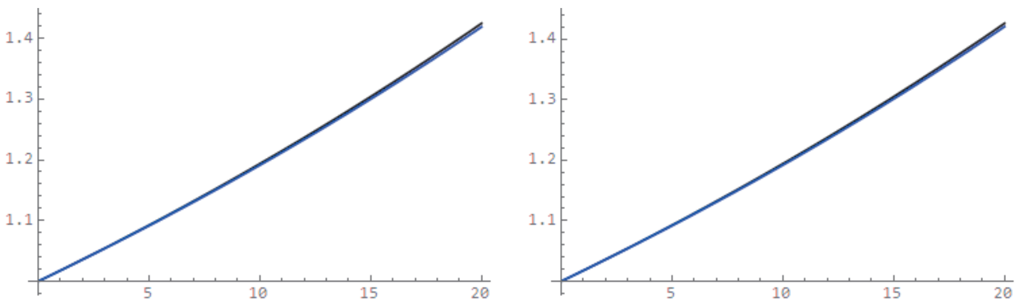


Figure 4: Effects of a shock on nominal interest rate i from 10% to 6%, with expenses paid only partially by cash.
Note: Thin black line represents initial steady-state values. Dashed line represents final steady-state values. If they coincide, initial and final steady-states coincide, despite the transitional dynamics.



(a) With total cash requirements (b) With partial cash requirements

Figure 5: Effects of a shock on nominal interest rate i from 10% to 6% in Consumption.
Note: Black line represents consumption with the policy. Blue line represents consumption without the policy.

positive effects of decreasing inflation. For instance, had we considered a drop in the nominal interest rate from 30% to 26%, we would obtain a 0.84% increase in welfare, which is even higher than in the baseline analysis. This means that the non-linear effect of inflation on growth induced by the non-linear *fee* can also be seen in transitional dynamics, explaining both positive and negative growth and welfare effects of an expansionary monetary policy according to different initial inflation rates. In particular, an

Table 1: Growth, allocation and Welfare Effects

Δg_Y	Δu_H	Δu_Q	Δu_n	Δu_x	$\Delta \mu$	$g_{Welfare}$
Alternative nonlinear education cost p function						
-0.007%	-0.10%	0.00%	0.00%	0.11%	-4%	-0.07%
Human Capital Depreciation ($\delta = 0.02$)						
0.022%	0.29%	0.02%	0.00%	-0.32%	-3.21%	2.24%
Population Growth Rate ($g_L = 0.01$) & ($\delta = 0.02$)						
0.021%	0.29%	0.00%	0.00%	-0.29%	-4.06%	8.60%
Population Growth and Altruism ($g_L = 0.01$; $m = 0.5$) & ($\delta = 0.02$)						
0.023%	0.29%	0.01%	0.00%	-0.27%	-4.04%	3.29%

Note: Δ is variation in the variable and g the growth rate of Welfare from a monetary policy that decreases the nominal interest rate from 10% to 6%.

expansionary monetary policy that departs from a low inflation level may imply negative growth and welfare effects while an expansionary monetary policy that departs from a low inflation level may imply relevant growth and welfare losses.

Secondly, we introduce a depreciation rate in the human capital accumulation, which has also been considered e.g. in Strulik (2005) and in Sequeira and Reis (2007), though in our case we consider a constant depreciation rate such that:

$$\dot{H} = \xi H_{H_t} - \delta_H H, \qquad \xi > 0. \tag{44}$$

Also in this case, we obtain growth and allocation effects similar to those in the baseline analysis. The crucial difference is on welfare, as it increases nearly 10 times to 2.24%. This due not only to the fact that initial welfare is much smaller (12.8 compared to 23.3 in the baseline analysis) but also due to transitional effects. In fact, in variations welfare increases nearly 0.15 in the baseline analysis and 0.29 in the model with human capital depreciation. The higher welfare effect (in variations) obtained in this exercise is solely due to transitional dynamics. First, the initial jump in consumption growth rates is higher, as well as they approach the new steady state faster from below.

Finally, we consider two changes in preferences also proposed by Strulik (2005), which are to consider population growth affecting the discounting of future consumption, as well as altruism affecting the discounting factor. In this case the utility function is changed to:

$$U = \int_0^\infty \frac{C_t^{1-\theta} - 1}{1-\theta} e^{-(\rho - m g_L)t} dt, \tag{45}$$

In this case we consider two alternatives: $m = 0$ where agents maximize *per capita* utility – they do not care about future generations, although population is growing ($g_L = 0.01$, i.e. population is growing at a 1% average rate), and $m = 0.5$ where agents are partially altruistic, caring about the utility of future generations in the dynasty. In the first of these cases, we obtain very similar growth and allocation effects but a quite higher welfare effect. Again this welfare effect is not seen only on growth rates but also on variations. In the second of these cases, altruism keeps the growth and allocation effects quite unchanged but decreases welfare effects due to the monetary policy when compared to the case with population growth but not altruism.

6 Conclusions

Monetary policy effects in the long-run growth rates are present in empirical results although most theoretical approaches are dominated by the superneutrality result, i.e., monetary policy does not have effects on the real side of the economy. This contradiction between empirical and theoretical results has seen an increasing interest from economists, due especially to the real oriented policy approaches some central banks have implemented following the subprime crisis or even before (e.g. in the Japanese case).

The study of the inflation-growth nexus within the endogenous growth framework has seen an increasing number of contributions in this decade. Those contributions tried to replicate a small but decreasing and non-linear effect of inflation on economic growth relying on R&D as the unique source of long-run growth. In fact, all contributions so far have ignored human capital accumulation as a source of growth and most of them have ignored the role of transitional dynamics and of reallocation of resources within the economy. We help to fill this gap in the endogenous growth literature with human capital accumulation, and horizontal and vertical R&D. We devise a model that includes monetary effects through a cash-in-advance constraint that affects both families and firms in the different sectors. We show that the mechanics to obtain the decreasing, small and non-linear effect of inflation on growth is due to the household funding of education.

Quantitatively, the model replicates both the small influence of monetary policy on growth but also highlights the effects it can have on welfare and allocations of resources throughout different sectors in the economy. We offer explanations for small (and also non-linear) effects of inflation on economic growth following monetary policies both on the transitional dynamics and on the long run. Our most interesting finding is that the empirical plausible effects of monetary policy on economic growth is due to the reallocation of resources from the goods production to the education sector.

It is worth noting that our results subsist for a number of robustness analysis on the crucial assumptions of the model. In fact, a depreciation in human capital and population growth would imply very similar effects on growth but higher effects on welfare when compared to the baseline scenario in which there are not depreciation in human capital nor population growth. More important than that, the non-linear fee for education proposed in this paper can also explain both positive and negative growth and welfare effects of an expansionary monetary policy according to different initial inflation rates. In particular an expansionary monetary policy that departs from a low inflation level may imply negative growth and welfare effects while an expansionary monetary policy that departs from a low inflation level may imply relevant growth and welfare losses.

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