Financial Literacy, Human Capital and Long-Run Economic Growth

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Abstract

We extend a two-sector endogenous growth model based on human capital accumulation along two different directions. First, by postulating that individuals may invest time-resources not only in the accumulation of human capital (general knowledge) but also in the accumulation of financial literacy (specific financial knowledge). Second, we maintain that the efficiency with which savings are transferred intertemporally may improve over time, e.g. through the presence of a financial system. We use the model to analyze the relationship between financial literacy and economic growth in the long run. We show that the properties of the balanced growth path equilibrium critically depend on how human capital and financial literacy affect the efficiency of the financial system. Moreover, finance promotes long-run economic growth through two alternative channels, driven either by dynamics of financial returns or by human capital accumulation, respectively. By calibrating the model to the US economy over the 1950-2019 period, we quantitatively assess the effect of financial literacy on long-term growth and the relative magnitude of the two channels.

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JEL Classification: G53, O40, O41

"...The recent crisis demonstrated the critical importance of financial literacy and good financial decision-making, both for the economic welfare of households and for the soundness and stability of the system as a whole. [...] The Federal Reserve recognizes that informed, educated consumers not only achieve better outcomes for themselves but, through careful shopping for and use of financial products, help to increase market efficiency and innovation." (Bernanke, 2011)

1 Introduction

The quick increase in the degree of financialization of the real economy, coupled with the advent of new and more complex financial instruments that can be used to consume, save and invest have amplified the need for

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more financial literacy.[1] In parallel, following the global financial crisis of 2007-2008 and the recent COVID-19 pandemic crisis, the debate on the importance of financial literacy has gained even more momentum as the least knowledgeable investors are considered the most exposed to financial and economic crunches.[2] Last but not least, financial market liberalizations and policy reforms have caused an ongoing shift in decision-making responsibility away from governments, institutions and employers towards private individuals. For all these reasons, the importance of having a sufficient level of financial literacy for individuals and households to take wiser decisions in various domains of their own businesses is by now widely recognized.

Financial literacy consists of a *"combination of awareness, knowledge, skill, attitude and behaviour necessary to make sound financial decisions and ultimately achieve individual financial wellbeing"* (OECD, 2018). Several papers extensively document the importance of financial literacy for economic and financial outcomes at the micro level (Guiso and Jappelli, 2008; Calvet et al., 2009; Christelis et al., 2010; Yoong 2011; Van Rooij et al., 2011; Behrman et al., 2012 Von Gaudecker, 2015; Clark et al., 2017; Deuflhard et al., 2019). For example, financial literates are less prone to over-indebtedness (Lusardi and Tufano, 2015; Lusardi and de Bassa Scheresberg, 2013; Lusardi et al., 2016) and to choose adjustable rate mortgages instead of less risky mortgages (Gathergood and Weber, 2017). They perform better in peer-to-peer lending markets (Chen et al., 2018) and choose mutual funds with lower fees (Hastings and Tejeda-Ashton, 2008). They are also more likely to plan for retirement and, as a result, to better allocate resources over their lifetimes (Goda et al., 2020). While the microeconomic literature has already reached a wide consensus on the importance of financial literacy, so far fewer contributions have investigated the possible macroeconomic effects of financial knowledge, and very limited are those, in particular, discussing its implications for long run economic growth.

Some works provide evidence of a positive link between financial literacy, savings decisions and wealth accumulation (Ameriks et al., 2003, Lusardi and Mitchell, 2007, and Jappelli and Padula, 2013). The most influential paper in this area is the one by Lusardi et al. (2017), who are the first to assess the importance of financial knowledge for wealth inequality. In particular, by using a stochastic life-cycle model that endogenizes the decision to acquire financial knowledge, Lusardi et al. (2017) show that gaps in financial literacy amplify differences in wealth accumulation patterns and the consequent perpetuation in wealth inequality. Along the same lines, Lo Prete (2013, 2018) tests how the ability to take advantage of different financial opportunities (measured by financial knowledge) may help reducing inequality across countries and over time.

To the best of our knowledge, Greenwood and Jovanovic's (1990) is the only work analyzing the impact of financial knowledge on economic growth. They demonstrate that financial institutions that encourage financial knowledge (by producing and diffusing, for example, better information on firms) induce a more efficient allocation of capital investment, and therefore promote both financial development and economic growth. However, there is no thorough theoretical work examining, within a dynamic setting, the channels through which financial literacy may ultimately affect long-run economic growth. Our paper aims at filling this important gap in the existing macroeconomic literature by analyzing whether and to what extent financial literacy is beneficial for long run macroeconomic outcomes.

We extend the Uzawa (1965) - Lucas (1988) multi-sector human capital-based endogenous growth model with the inclusion of finance, which in our framework transfers savings intertemporally. We also provide the agents with the possibility to invest in financial literacy. Financial literacy does not represent an input

[1] For the sake of simplicity, in the present paper we shall use such terms as financial literacy and financial knowledge interchangeably. For a more systematic review of the possible definitions and synonyms of financial literacy in the literature, see Lusardi and Mitchell (2007) and more recently Goyal and Kumar (2021, pp. 80-81).

[2] See, among others, Gerardi et al. (2010), Hasler et al. (2018), Brown et al. (2016a), Guiso and Viviano (2015). Feng et al. (2019) show that households with insufficient financial knowledge are more financially vulnerable because they are more likely both to have fewer assets and to choose high-cost unsecured debts. By contrast, financial knowledge is found to enable individuals, for example, to plan for wealth accumulation (Ameriks et al. 2003), to be more financially included (Grohmann et al. 2018), and to choose investments that are the most suitable for their own needs (Bianchi 2018).

in the production of the final good, but positively influences the efficiency with which, e.g. through the financial system, currently available savings are transformed in future (physical) capital. We postulate that this efficiency – measured by the return on households' savings – ultimately depends on the macroeconomic conditions of the economy, and is therefore determined endogenously in the model.[3]

While financial literacy and human capital are not synonyms, they are strongly related to each other (Lusardi and Mitchell 2008; Cole et al., 2011; Arrondel, 2018). Individuals can make use of relatively basic economic or mathematical concepts generally acquired during their years of study to make appropriate financial decisions (Mandell, 2008; Al-Bahrani et al., 2020). With this in mind, our model builds on the idea that, like human capital, financial literacy is an intangible asset that forward-looking agents can accumulate over time. This explains why a macroeconomic theory of financial literacy needs to be embedded within a human-capital based multi-sector endogenous growth setting.

In our framework human capital is accumulated by purposefully investing time (i.e., man-hours) in general education. Human capital investment therefore competes for the same scarce input (time) with the acquisition of new financial literacy, and with the use of human capital for production purposes. Thanks to this feature, we obtain that, from a macroeconomic perspective, the investment in financial education gives rise to a dynamic cost/benefit trade-off. On the benefit-side, a higher level of financial literacy allows households to obtain higher returns on their savings, or asset holdings (as in Lusardi et al., 2017, and Jappelli and Padula, 2013). On the cost-side, the investment in financial education increases the opportunity cost of time for human capital accumulation (as in Kim et al., 2016). The consequences of this trade-off drive the main results of our model.

We first show that accounting for the presence of finance as a mechanism that transfers savings intertemporally in an otherwise standard multi-sector, human capital-based endogenous growth model dramatically changes the properties of the balanced growth path (BGP, hereafter) equilibrium. Indeed, we find that if the return on savings provided by the financial system grows at a strictly positive rate, along the BGP the key variables grow at constant but different rates. Physical capital grows faster than production and consumption, which in turn grow faster than human capital and the stock of financial literacy. This implies that the relative size of different economic sectors changes over time.

Our second result consists in the identification of the two distinct channels through which finance and financial literacy can benefit economic growth. In addition to the "human capital channel", that works through a change in the overall rate of growth of human capital in the economy, the second one (the "financial return channel") operates exactly through an increase in the efficiency with which savings may be transferred across time. To provide an intuitive overview of these two channels, first of all notice that in our model, along the BGP equilibrium, economic growth depends positively on both the growth rate of human capital and on the growth rate of financial returns (as already defined). Suppose that the return of finance relies more strongly on financial literacy formation than on the accumulation of human capital. In this case, the following trade-off takes place. On the one hand, the higher opportunity cost of time devoted to human capital accumulation (as opposed to financial knowledge accumulation) leads to a lower growth rate of human capital, relatively to an analogous economy without finance. Ceteris paribus, this lowers the overall rate of economic growth, as well. On the other hand, given that the financial return is more sensitive to financial education, forward-looking agents will decide to accumulate financial literacy faster as, by doing so, they will be able to earn higher financial returns in the future. Provided that finance is particularly responsive to the investment in financial knowledge accumulation, the second effect more than compensates for the first one, ultimately leading the growth rate of the economy to exceed the one that can be finally obtained in an analogous model without finance. In other words, even if financial knowledge accumulation drains resources away from general education and therefore may reduce long run growth, such an effect can

[3]In our model finance is formally introduced as an efficiency-function depending on several macroeconomic variables. For the sake of simplicity, from now on, such terms as finance or financial system will be employed indistinctly. Hence, finance/financial system are defined in the paper in a very broad sense.

be compensated by the improved financial efficiency, which is due to the higher level of financial literacy available in the economy.

Conversely, the human capital channel is relevant when the financial return is relatively more sensitive to human capital accumulation than to the investment in financial education. In this case the presence of finance induces to accumulate human capital faster, i.e., to devote more time to general education, than in an analogous multi-sector endogenous growth model. The higher growth rate of human capital in turn increases economic growth.

In order to assess the quantitative effect of financial literacy on long-term growth we calibrate our model by using data on the US economy over the 1950-2019 period. Based on well-established empirical findings, we postulate that the positive effect of human capital and financial literacy on the return on savings generated by the financial system is due to the increased stock market participation. By now, there is large empirical evidence according to which stock market participation has increased capital market efficiency in the US (e.g. Greenwood and Jovanovic, 1990, and Bencivenga and Smith, 1991, among others), and that both the education level (human capital) and financial literacy are positively related to stock market participation (see for example Campbell, 2006, van Rooij et al., 2011 and 2012, Cole and Shastry, 2009, among others).

Our theoretical model is able to match the observed growth rates of both human capital and per-capita income in the US for very high levels of the elasticities of financial returns with respect to financial literacy, and to the investment in financial knowledge accumulation. These results are comparable with van Rooij et al.'s (2011), who find a strong empirical effect of both formal education and financial literacy on stock market participation. Our baseline calibration also shows that both elasticities are larger than one, suggesting that even a small increase in financial literacy may generate sizeable effects on economic growth in the long run by inducing higher stock market participation. Given that the financial returns are positive in the long run, higher stock market participation increases long-term growth by making more resources available for production.

In the baseline scenario of our calibration the financial return channel prevails over the human capital one, suggesting that the positive effect of finance on economic growth is mainly driven by the financial return dynamics, rather than by human capital formation. However, any channel might prevail depending on the magnitude of two fundamental parameters of the model, namely the degree of agents' (im)patience and the productivity of human capital in the production of new human capital.

The paper is structured as follows. In section 2 we introduce the model. In section 3 we discuss our main results by focusing on the long-run BGP equilibrium predictions of our setup. Section 4 compares the growth rate of our economy with the one in a standard multi-sector model without finance. Section 5 presents our calibrations, quantitatively assessing the relative importance of the financial return and human capital channels. Section 6 concludes and proposes possible paths for future research. Some sensitivity analysis for our calibration and the proofs of our main results are presented in appendix A and B, respectively.

2 The Model

We analyze a discrete-time endogenous growth model *à-la* Uzawa-Lucas (1988) extended for the inclusion of finance.[4] For the sake of simplicity we abstract from population growth and capital depreciation. The social planner seeks to maximize social welfare subject to the evolution of physical capital, human capital and financial literacy by determining consumption and the shares of human capital to employ in the production of the final consumption good and in financial education. While human capital quantifies general productivity-augmenting skills (i.e., education, cognitive ability and numeracy skills), financial literacy measures skills

[4]Few works have developed similar three-sector extensions of the Uzawa-Lucas model to account for the accumulation of knowledge (La Torre and Marsiglio, 2010; La Torre et al., 2015). Different from them, in our setting the third sector produces an intangible asset (namely, financial literacy), which does not represent an input in the production of the final consumable good.

specific to finance which do not contribute to the aggregate production. The final consumption good is produced by employing physical capital and an endogenously determined share of human capital. Physical capital accumulation depends on saving augmented for the efficiency of the financial system, which in turn depends on macroeconomic conditions and in particular on income, human capital, and financial literacy (both in terms of stock and investment flow). Financial literacy is produced by combining existing financial literacy and an endogenously determined share of human capital. Human capital formation instead depends on the remaining share of human capital not employed in other sectors.

Our model builds on two major assumptions, related to the return on saving generated by the financial system, and to the drivers of financial literacy acquisition, respectively.

The first assumption hinges on the idea that, at the macroeconomic level, the return on agents' savings may be influenced not only by their degree of financial literacy but also by their human capital stock, together with their investment in these two skills. Indeed, households with a higher level of financial knowledge pay lower costs to collect and process information and obtain higher returns on their savings (as in Lusardi et al., 2017, Jappelli and Padula, 2013, and similarly to Arrow, 1987). Moreover, better educated households are more likely to own stock and to earn positive abnormal returns on their financial portfolio (Campbell, 2006, and Cole and Shastry, 2008). Kim et al. (2016) also relate the returns on savings with the degree of financial literacy, and share an important feature with our setup. They build a life-cycle dynamic model where the time required to manage one's financial portfolio is traded-off with the time devoted to developing job-specific human capital. This is similar to the key trade-off in our model, where human capital accumulation competes with financial knowledge accumulation for the same scarce input, i.e. time. Two relevant features make our paper especially different from Kim et al.'s (2016) contribution. The first difference rests in the aim of the paper, since they try to explain "investor inertia", that is the tendency to maintain one's investment portfolio for very long periods of time, a possible symptom of financial illiteracy. The second major difference has to do with the modelling of the process of investment in both human capital and financial literacy. Whereas their notion of human capital is informed by the idea of job-specific-skills accumulated by working (as in Becker, 1964), in our paper human capital is not accumulated on-the-job, but rather by purposefully investing time (i.e., man-hours) in general education (as in Uzawa-Lucas).

Our second key assumption postulates that financial literacy requires two inputs to be augmented: formal education (i.e., human capital) and the stock of specialized financial knowledge already existing.[5] Indeed, several studies (e.g. Delavande et al., 2008) argue that financial knowledge is self-productive, in the sense that greater initial financial knowledge contributes to enhance the efficiency with which new financial knowledge is obtained over time. In addition, there is a large evidence that financial knowledge is closely related to numeracy skills. Mandell (2008), and Al-Bahrani et al. (2020) find that financial literacy and education are correlated since the early stages of the life-cycle. Also, individuals with higher levels of education are the most likely to be financially literate (Lusardi and Mitchell 2008; Cole et al., 2011), and schooling increases the probability of financial market participation and investing in the stock market (Cole and Shastry, 2008; Thomas and Spataro, 2018). These results suggest that general knowledge (education) and specialized knowledge (financial literacy) both contribute positively to effective financial decision-making. Together, all these pieces of evidence clearly show that both the existing amount of financial knowledge and the number of man-hours purposely devoted to the production of new financial literacy are two essential factors in the process of raising the degree of financial knowledge of a population.

We can now describe in major details the structure of our model economy. Social welfare is the sum of the instantaneous utilities of structurally-identical individuals of measure one, discounted at the exogenous factor $\beta \in (0, 1)$. The utility function of the representative agent depends only on consumption $c_t > 0$ and

[5] The OECD (2020) defines financial education as*: "...The process by which financial consumers/investors improve their understanding of financial products, concepts and risks and, through information, instruction and/or objective advice, develop the skills and confidence to become more aware of financial risks and opportunities, to make informed choices, to know where to go for help, and to take other effective actions to improve their financial wellbeing."*

takes a logarithmic form, $u(c_t) = \ln(c_t)$. The unique final consumption good $y_t > 0$ can be either consumed or saved and invested in physical capital accumulation. It is produced through a Cobb-Douglas technology as follows:

$$y_t = k_t^\alpha (u_t h_t)^{1-\alpha} \tag{1}$$

where $k_t > 0$ denotes physical capital, $h_t > 0$ human capital, $a_t > 0$ financial literacy, $0 < u_t < 1$ the share of human capital devoted to output production, while $0 < \alpha < 1$ is the physical capital share. Also new financial knowledge is produced through a Cobb-Douglas technology (Delavande et al., 2008), as follows:

$$a_{t+1} = (\nu_t h_t)^{1-\xi} a_t^\xi \tag{2}$$

where $0 < \nu_t < 1$ denotes the share of human capital devoted to financial education and $0 < \xi < 1$ measures the elasticity of financial literacy production with respect to the existing stock of financial knowledge. New human capital is produced through a linear technology:

$$h_{t+1} = b(1 - u_t - \nu_t)h_t \tag{3}$$

where $b > 1$ measures the productivity of education in human capital formation. Note that while human capital is an input in the production of the final good, financial literacy is not and thus it has no effect on output (see (1)).

In our model finance transfers savings intertemporally. In particular, it is broadly identified with an efficiency function $R(.)$ that determines how many units of physical capital are obtained at time $t + 1$ from one unit of saving at time t.

For the moment, we do not take any stand concerning the specific form that the function $R(.)$ can take. We just allow it to be neoclassical, i.e. everywhere continuous and differentiable, strictly increasing and concave in all its arguments. We also consider that $R(\cdot)$ depends on output y_t and on financial literacy, both in the form of stock a_t and flow ν_t. Accordingly, we write $R(y_t, a_t, \nu_t)$. It follows that the dynamic evolution of physical capital reads as:

$$k_{t+1} = R(y_t, a_t, \nu_t)(y_t - c_t),$$

which, from (1), can be also recast as:

$$k_{t+1} = R(k_t, h_t, a_t, \nu_t, u_t)(y_t - c_t) \tag{4}$$

Note that if $R = 1$ for all inputs $(k_t, h_t, a_t, \nu_t, u_t)$, then our model reduces to a standard multisector endogenous growth model (see for example La Torre et al., 2015).

Equations (1), (3) and (4) show that the allocation of a share ν_t of human capital to financial education involves a trade-off. On the one hand a higher ν increases the return on capital invested and therefore increases the future stock of physical capital. On the other hand it also reduces the amount of human capital that can be devoted to production and to the acquisition of new human capital. Therefore, the macroeconomic impact of financial education on long run economic growth is not obvious *a priori.*

Given the initial conditions $k_0 > 0$, $h_0 > 0$ and $a_0 > 0$, the social planner's needs to choose consumption and the share of human capital to allocate across sectors in order to maximize social welfare. Her optimization problem can thus be summarized as follows:

$$\begin{aligned}
\max_{\{c_t,u_t,\nu_t\}_{t=0}^{\infty}} \quad & \sum_{t=0}^{\infty} \beta^t \ln(c_t) \\
s.t. \quad & k_{t+1} = R(k_t, h_t, a_t, u_t, \nu_t)(y_t - c_t) \\
& h_{t+1} = b(1 - u_t - \nu_t)h_t \\
& a_{t+1} = (\nu_t h_t)^{1-\xi} a_t^{\xi} \\
& k_0, h_0, a_0 > 0
\end{aligned} \tag{5}$$

The above problem clearly shows that both finance and financial literacy may play a crucial role in determining macroeconomic outcomes. In particular, $R(\cdot)$ affects the returns on savings, and therefore physical capital accumulation. There exist several channels through which finance may affect the aggregate return on savings. Financial intermediaries are able to better funnel savings to firms (Pagano, 1993), allow agents to create diversified portfolios with higher expected returns (King and Levine, 1993), improve the access to education (De Gregorio, 1996). At the macroeconomic level, the financial system allocates aggregate saving to the productive sector. If financial intermediaries improve the allocation of capital (Greenwood and Jovanovic, 1990), and increase the efficiency of investment (Bencivenga and Smith, 1991), then they earn positive returns on capital invested, as it has been observed empirically over sufficiently long periods. All these theoretical explanations suggest that R may be higher than one, at least for some values of $(k_t, h_t, a_t, u_t, \nu_t)$.

3 The Long Run Equilibrium

After some algebra (see Appendix B), it is possible to obtain a closed form solution of problem (5), and to derive explicitly the optimal policy and the optimal dynamics of physical capital, human capital and financial literacy. This result is summarized in the next proposition.

Proposition 1. *Let $\varepsilon_{R,i} \geq 0$ with $i = \{k, h, a, u, \nu\}$ denote the elasticity of the function R with respect to variable i. If $\varepsilon_{R,k} \leq \frac{1-\alpha}{\beta}$ then:*

i) The optimal policy rules for consumption c_t, for the share of human capital allocated respectively to the production of final good u_t, and to financial education ν_t are given by:

$$c_t = \frac{1 - \alpha\beta - \beta\varepsilon_{R,k}}{1 - \beta\varepsilon_{R,k}} k_t^{\alpha} \overline{u}^{1-\alpha} h_t^{1-\alpha} > 0 \tag{6}$$

$$u_t = \overline{u} = \frac{1 - \beta\Theta}{\Delta} \in (0, 1) \tag{7}$$

$$\nu_t = \overline{\nu} = \frac{1 - \beta\Theta}{\Delta} \frac{\varepsilon_{R,\nu} + \frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}}{\varepsilon_{R,u} + \frac{1-\alpha}{\alpha\beta}\left(1 - \beta\varepsilon_{R,k}\right)} \in (0, 1) \tag{8}$$

where:

$$\Theta = \frac{1-\alpha-\beta\varepsilon_{R,k}+\alpha\beta\left(\varepsilon_{R,k}+\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)}{1-\alpha-\beta\varepsilon_{R,k}+\alpha\beta\left(\varepsilon_{R,k}+\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)+\alpha\beta(1-\beta)\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}-\varepsilon_{R,h}\right)} \tag{9}$$

$$\Delta = \frac{\varepsilon_{R,u}+\frac{1-\alpha}{\alpha\beta}\left(1-\beta\varepsilon_{R,k}\right)+\varepsilon_{R,\nu}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}}{\varepsilon_{R,u}+\frac{1-\alpha}{\alpha\beta}\left(1-\beta\varepsilon_{R,k}\right)} \tag{10}$$

ii) The optimal dynamics of physical capital k_{t+1}, human capital h_{t+1} and financial literacy a_{t+1} are given *by:*

$$k_{t+1} = R\frac{\alpha\beta}{1-\beta\varepsilon_{R,k}}k_t^{\alpha}\overline{u}^{1-\alpha}h_t{}^{1-\alpha} \tag{11}$$
$$h_{t+1} = b(1-\overline{u}-\overline{\nu})h_t \tag{12}$$
$$a_{t+1} = \overline{\nu}^{1-\xi}h_t^{1-\xi}a_t^{\xi} \tag{13}$$

Proposition 1 derives explicitly the optimal policy (point (i)) and the optimal dynamics of physical capital, human capital and financial literacy (point (ii)). For these policies and dynamics to be well defined the elasticity of the financial system's return with respect to physical capital $\varepsilon_{R,k}$ needs to be sufficiently small, while no restrictions on the magnitude of other elasticities are required.[6] With R taking a neoclassical form, these elasticities are all constant and non-negative. The magnitude of such elasticities plays a critical role in determining the optimal policy rules.

If the efficiency of the financial system is such that $R = 1$ for all possible values of k, h, a, u, ν, then all elasticities $\varepsilon_{R,i}$ are equal to zero. In this case it is optimal not to invest in financial education (i.e., $\overline{\nu} = 0$). Given that financial literacy does not contribute to the production of the final good and generates no economic benefits (see (1)), investing in financial knowledge represents only a costly diversion of resources. It follows then that the optimal policies and dynamics in Proposition 1 reduce to those in a standard multisector endogenous growth model: the share of human capital devoted to production equals $\overline{u} = 1-\beta$, and consumption represents a fixed quota of production, $c_t = (1-\alpha\beta)y_t$.

If instead R is larger than one, at least for some values of k, h, a, u, and ν, then finance and financial literacy affect the optimal policies and dynamics both in the short run and in the long run. In this case investing in financial education does not involve only a current cost but also a future benefit due to the increased resources generated by the financial system. It is then optimal to allocate a strictly positive share of human capital to acquire new financial knowledge (i.e. $\overline{\nu} > 0$).

In particular, we show (see the proof of the proposition in Appendix B) that in the short run the optimal shares of human capital allocated to output production and to financial education ($\overline{u}$ and $\overline{\nu}$ respectively) both increase with their own return elasticity and decrease with the return elasticity with respect to human capital (i.e., $\frac{\partial\overline{u}}{\partial\varepsilon_{R,u}} > 0$, $\frac{\partial\overline{\nu}}{\partial\varepsilon_{R,\nu}} > 0$, $\frac{\partial\overline{u}}{\partial\varepsilon_{R,h}} < 0$ and $\frac{\partial\overline{\nu}}{\partial\varepsilon_{R,h}} < 0$). These results are intuitive. It is optimal to allocate a larger share of human capital to a certain activity whenever the elasticity of the financial return with respect to such activity increases. Similarly, if the financial return becomes relatively more sensitive to human capital then it is optimal to accumulate more human capital, at the cost of reducing the share of human capital devoted to output production and to financial education. The effects of $\varepsilon_{R,a}$ on $\overline{u}$ and $\overline{\nu}$ are instead ambiguous, as they depend on the relative magnitude of $\varepsilon_{R,u}+\varepsilon_{R,\nu}$ and $\varepsilon_{R,h}$: both $\overline{u}$ and $\overline{\nu}$ increase (decrease) with $\varepsilon_{R,a}$ whenever $\varepsilon_{R,u}+\varepsilon_{R,\nu} > \varepsilon_{R,h}$ (whenever $\varepsilon_{R,u}+\varepsilon_{R,\nu} < \varepsilon_{R,h}$).[7]

The discussion above suggests that the relative size of $\varepsilon_{R,u}+\varepsilon_{R,\nu}$ and $\varepsilon_{R,h}$ plays an important role in determining the incentives to accumulate human capital vis-à-vis producing output and accumulating financial literacy. In order to quantify this trade-off, it is useful to start from the specific case in which $\varepsilon_{R,u}+\varepsilon_{R,\nu} = \varepsilon_{R,h}$. Under this condition, from (9) we have that $\Theta = 1$ and $\overline{u}+\overline{\nu} = 1-\beta$.[8] Human capital grows at the rate $b\beta$, so that the presence of a financial system does not affect the human capital dynamics (see (3)).[9] The analysis of this special case allows us to clarify the role of the existing level of financial literacy on the optimal allocation of resources. Whenever $\varepsilon_{R,u}+\varepsilon_{R,\nu} = \varepsilon_{R,h}$, provided that $\varepsilon_{R,a} > 0$, it is optimal to allocate a positive quota of the current stock of human capital to financial education. At

[6] If the elasticity $\varepsilon_{R,k} > \frac{1-\alpha\beta}{\beta}$ the solution of the model exhibits negative consumption. The condition $\varepsilon_{R,k} \leq \frac{1-\alpha}{\beta} < \frac{1-\alpha\beta}{\beta}$ is sufficient to ensure that $\overline{u} > 0$, $\overline{\nu} > 0$ and that $\overline{u}+\overline{\nu} < 1$ (see the proof of Proposition 1 in Appendix B for details).

[7] Intuitively, the higher $\varepsilon_{R,a}$, the higher the incentive to accumulate financial literacy. Given (2), this can be done in two ways, either by increasing $\overline{\nu}$ or by investing more in human capital formation. If $\varepsilon_{R,u}+\varepsilon_{R,\nu} > \varepsilon_{R,h}$, then the first choice is optimal, given that a higher $\overline{\nu}$ has a stronger effect on R than an increase in human capital.

[8] For this result, see also the proof of Proposition 1 in Appendix B.

[9] This result holds irrespective of the effect of financial literacy on R, i.e. for any $\varepsilon_{R,a} \geq 0$ and $\varepsilon_{R,\nu} \geq 0$.

the same time, the quota of human capital used for output production is reduced in such a way that the overall amount of human capital devoted to human capital formation is equal to the one that can be found within a framework without finance. Instead, if $\varepsilon_{R,u} + \varepsilon_{R,\nu} > \varepsilon_{R,h}$, then $\overline{u} + \overline{\nu} > 1 - \beta$, and the rate of growth of human capital is lower than $b\beta$. The presence of a financial system that earns returns highly sensitive to financial education makes it optimal to devote a relatively large amount of human capital to this task, partially offsetting the accumulation of new human capital. Conversely, if $\varepsilon_{R,u} + \varepsilon_{R,\nu} < \varepsilon_{R,h}$ then $\overline{u} + \overline{\nu} < 1 - \beta$ and human capital grows at a rate higher than $b\beta$. The financial system provides incentives to further accumulate human capital. These observations are crucial to understand the effect of finance and financial literacy on long-term economic growth.

In the next proposition we characterize the long-run outcome of our economy, which is represented by a BGP equilibrium, defined as the particular situation where the rates of growth of variables remain constant (Lucas, 1988).

Proposition 2. *Let $\varepsilon_{R,k} = 0$. If $b > \frac{1}{\beta\Theta}$ then along the BGP the shares of human capital allocated to production and to financial literacy accumulation are constant, such that:*

$$\overline{u} = \frac{1 - \beta\Theta}{\Delta} \in (0, 1) \tag{14}$$

$$\overline{\nu} = \frac{1 - \beta\Theta}{\Delta} \frac{\varepsilon_{R,\nu} + \frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}}{\varepsilon_{R,u} + \frac{1-\alpha}{\alpha\beta}} \in [0, 1) \tag{15}$$

and the growth rates of consumption (γ_c), physical capital (γ_k), human capital (γ_h), financial literacy (γ_a) and production (γ_y) are constant and given by:

$$\gamma_h = \gamma_a = b\beta\Theta - 1 > 0 \tag{16}$$

$$\gamma_k = (1 + \gamma_R)^{\frac{1}{1-\alpha}}(1 + \gamma_h) - 1 > 0 \tag{17}$$

$$\gamma_y = \gamma_c = (1 + \gamma_R)^{\frac{\alpha}{1-\alpha}}(1 + \gamma_h) - 1 > 0, \tag{18}$$

where $\gamma_R \geq 0$ is the growth rate of the financial system's return.

Provided that the elasticity of the efficiency function R with respect to physical capital is null (i.e., $\varepsilon_{R,k} = 0$), Proposition 2 explicitly determines the growth rates of the main variables along the BGP equilibrium. If the productivity of human capital in the creation of new human capital is large enough (i.e., $b > \frac{1}{\beta\Theta}$), then all the state variables grow at a strictly positive and constant rate. The shares of human capital allocated to output production and to financial education are instead constant (see Proposition 1). Proposition 2 illustrates two key results.

First, (16)-(18) imply that whenever R is constant over time (i.e., $\gamma_R = 0$), physical and human capital, consumption, production and financial literacy all grow at the same common rate, given by (16), along the BGP. If instead R grows at a strictly positive rate (i.e., $\gamma_R > 0$), this introduces a wedge between the growth rates of the main variables. Specifically, physical capital grows faster than production and consumption, which in turn grow faster than human capital and the stock of financial literacy. The fact that variables grow at different rates at the BGP implies that over time the relative size of different economic sectors will change.

Second, from (18) it is clear that the financial system, and the accumulation of financial literacy, impact long run economic growth as long as they affect the growth rate of human capital (γ_h) and/or that of the financial system's return (γ_R). Indeed, these growth rates depend on both a and ν. In particular, since along the BGP the investment in financial education is constant, the stock of financial literacy grows at a positive, constant rate (γ_a). All else being equal, the faster the growth of a, the faster the growth of R, and the higher γ_y. Therefore, accumulating financial literacy (i.e. investing in financial education) may benefit economic growth in the long run, provided that financial literacy and financial education improve the efficiency of the financial system (this is the "financial return" channel). But there is also another way through which the financial system is potentially relevant for long-run growth. As we discussed above in the analysis of the case $\varepsilon_{R,u} + \varepsilon_{R,\nu} < \varepsilon_{R,h}$ after Proposition 1, the financial system might increase the accumulation of human capital, even if individuals invest in financial education. If this is the case, then the long-run growth of an economy with a financial system is amplified by the faster accumulation of human capital (this is the "human capital" channel).

In the proof of the proposition (see Appendix B) we show that the human capital growth rate γ_h increases with $\varepsilon_{R,h}$ and decreases with $\varepsilon_{R,u}$ and $\varepsilon_{R,\nu}$, while it increases (decreases) with $\varepsilon_{R,a}$ if $\varepsilon_{R,u} + \varepsilon_{R,\nu} > \varepsilon_{R,h}$ (if $\varepsilon_{R,u} + \varepsilon_{R,\nu} < \varepsilon_{R,h}$). These results are intuitive, and to a large extent mimic what we have already discussed earlier in relation to the effects of the elasticities of the the efficiency term R with respect to the optimal allocation shares of human capital. The higher the elasticity of R with respect to human capital, the higher the incentive to invest in human capital formation and thus the faster its growth ($\frac{\partial \gamma_h}{\partial \varepsilon_{R,h}} > 0$). Conversely, when the elasticities $\varepsilon_{R,u}$ and $\varepsilon_{R,\nu}$ increase, it becomes optimal to allocate a larger share of human capital to production and to financial education so that γ_h decreases ($\frac{\partial \gamma_h}{\partial \varepsilon_{R,u}} < 0$ and $\frac{\partial \gamma_h}{\partial \varepsilon_{R,\nu}} < 0$).[10] Under some regularity conditions on the function R, provided that the cross elasticities of the return function with respect to the allocation shares are positive (i.e., $\frac{\partial \overline{u}}{\partial \varepsilon_{R,\nu}} > 0$, and $\frac{\partial \overline{\nu}}{\partial \varepsilon_{R,u}} > 0$), then the effects of the elasticities $\varepsilon_{R,h}$, $\varepsilon_{R,u}$, $\varepsilon_{R,\nu}$ and $\varepsilon_{R,a}$ on human capital growth extend also to output growth, i.e. $\frac{\partial \gamma_y}{\partial \varepsilon_{R,h}} > 0$, $\frac{\partial \gamma_y}{\partial \varepsilon_{R,u}} < 0$, $\frac{\partial \gamma_y}{\partial \varepsilon_{R,\nu}} < 0$, while the sign of $\frac{\partial \gamma_y}{\partial \varepsilon_{R,a}}$ is ambiguous, being positive if $\varepsilon_{R,u} + \varepsilon_{R,\nu} > \varepsilon_{R,h}$ and negative otherwise.

4 The Role of the Financial System and of Financial Literacy

We now compare our results with those arising from the multi-sector growth model based on the sole accumulation of human capital by Uzawa-Lucas (UL hereafter). In order to make a meaningful comparison we consider that all structural parameters are the same across the two setups, with the exception of those related to financial activities which are not present in the UL model.

In the UL framework all variables grow at the same constant rate γ^{UL} along the BGP, and the share of human capital allotted to production $\overline{u}^{UL}$ is constant (Lucas, 1988):

$$\gamma^{UL} = b\beta - 1 \tag{19}$$
$$\overline{u}^{UL} = 1 - \beta, \tag{20}$$

The BGP is well defined, i.e., $\gamma^{UL} > 0$ and $0 < \overline{u}^{UL} < 1$, whenever $b > \frac{1}{\beta}$ (see La Torre et al., 2015). Note that such a condition does not necessarily ensure sustained growth in our finance-extended model (see (16) and (18)). Therefore, in the following we shall restrict our analysis to the parameter region $b > \max\{\frac{1}{\beta\Theta}, \frac{1}{\beta}\}$. Under this condition, both frameworks are characterized by endogenous growth and their respective BGPs are well defined. For easiness of comparison we also assume that the productivity of education in human capital formation b is the same across the two setups. The next proposition compares the growth rates in the two models.

Proposition 3. *Let $\varepsilon_{R,k} = 0$. Consider an UL framework with γ^{UL} as in (19) and $\overline{u}^{UL}$ as in (20), and our model described in (5). Assume that $b > \max\{\frac{1}{\beta\Theta}, \frac{1}{\beta}\}$ is the same in the two setups. Then along the BGP the following results hold:*

(i) If $\varepsilon_{R,u} + \varepsilon_{R,\nu} \geq \varepsilon_{R,h}$ then $\gamma_h \leq \gamma^{UL}$ and $\gamma_y \geq \gamma^{UL}$ provided that $1 + \gamma_R \geq (\frac{1}{\Theta})^{\frac{1-\alpha}{\alpha}}$;

(ii) If $\varepsilon_{R,u} + \varepsilon_{R,\nu} < \varepsilon_{R,h}$ then $\gamma_h > \gamma^{UL}$ and $\gamma_y > \gamma^{UL}$.

[10]The effect of the elasticity $\varepsilon_{R,a}$ on the human capital growth rate γ_h instead is ambiguous: $\frac{\partial \gamma_h}{\partial \varepsilon_{R,a}} > 0$ if $\varepsilon_{R,u} + \varepsilon_{R,\nu} > \varepsilon_{R,h}$, otherwise $\frac{\partial \gamma_h}{\partial \varepsilon_{R,a}} < 0$ when $\varepsilon_{R,u} + \varepsilon_{R,\nu} < \varepsilon_{R,h}$.

Proposition 3 states that the size of the elasticities of the financial system's return with respect to the human capital stock and its allocation determines whether finance (i.e., $R > 1$) can benefit or not long-run economic growth. Specifically, it identifies two distinct channels through which this may occur, described respectively by points (i) and (ii) in Proposition 3.

When $\varepsilon_{R,u} + \varepsilon_{R,\nu} \geq \varepsilon_{R,h}$ (point (i)) the beneficial effect of finance on economic growth occurs through the "financial return channel". In this case the efficiency of the financial system depends more strongly on financial education and output production efforts than on the human capital stock. This makes convenient to devote less time to general education, relatively to an analogous economy without a financial system (i.e., $R = 1$), leading to a lower growth rate of human capital γ_h and potentially to a lower rate of growth γ_y. However, if the financial system is particularly effective in intertemporally transferring savings, it can more than compensate for this adverse effect, leading the economic growth rate to exceed the one in the UL model. More precisely, if the growth rate of R is sufficiently high (i.e., $1 + \gamma_R \geq (\frac{1}{\Theta})^{\frac{1-\alpha}{\alpha}} > 1$), then the increase in the efficiency with which finance is able to transfer savings over time may more than compensate for the detrimental effects induced by the lower human capital accumulation. If this is the case, then the long-run growth rate of the economy exceeds the one that would be achieved without a financial system. The ultimate effect of financial literacy on long run growth therefore depends on its impact on the efficiency of the financial system.

Instead, when $\varepsilon_{R,u} + \varepsilon_{R,\nu} < \varepsilon_{R,h}$ (point (ii)) the presence of a financial system generates beneficial effects on growth through the "human capital channel". In this case the efficiency of the financial system depends very strongly on human capital so that it is optimal to devote more time to general education than in the model without a financial system. This leads to a growth rate of human capital higher than in UL. The higher rate of human capital growth in turn promotes faster economic growth.

As a priori we do not have information about the relative size of the elasticities of the efficiency term R, at this stage both the "financial return" and the "human capital" channels are equally viable alternative explanations of the determinants of long run economic growth. In order to disentangle the relative strength of these two effects, we need to determine explicitly the growth rate of output γ_y. Given that γ_y depends on the growth rate γ_R, at this stage we need to specify the particular functional form that the function R takes, as we formally do in the next example.

Example 1. *Let the function R be Cobb-Douglas:*

$$\begin{aligned} R_t &= (u_t h_t)^{\varepsilon_u} (\nu_t h_t)^{\varepsilon_\nu} h_t^{\chi} a_t^{\varepsilon_a} \\ &= u_t^{\varepsilon_u} \nu_t^{\varepsilon_\nu} h_t^{\varepsilon_u + \varepsilon_\nu + \chi} a_t^{\varepsilon_a} \end{aligned} \tag{21}$$

where $\varepsilon_i \geq 0$ denote the elasticities of R with respect to $i = \{u, \nu, a\}$ respectively, and $\varepsilon_h = \varepsilon_u + \varepsilon_\nu + \chi$. Under the conditions of Proposition 2, along the BGP we have:

$$\frac{R_t}{R_{t-1}} = 1 + \gamma_R = \frac{\overline{u}^{\varepsilon_u} \overline{\nu}^{\varepsilon_\nu} h_t^{\varepsilon_h} a_t^{\varepsilon_a}}{\overline{u}^{\varepsilon_u} \overline{\nu}^{\varepsilon_\nu} h_{t-1}^{\varepsilon_h} a_{t-1}^{\varepsilon_a}} = \left(\frac{h_t}{h_{t-1}}\right)^{\varepsilon_h} \left(\frac{a_t}{a_{t-1}}\right)^{\varepsilon_a} = (1 + \gamma_h)^{\varepsilon_h + \varepsilon_a}$$

because $\gamma_a = \gamma_h = b\beta\Theta - 1$ (see (16)). From (18), the growth rate of output is ultimately given by:

$$1 + \gamma_y = (b\beta\Theta)^{1 + \frac{\alpha(\varepsilon_h + \varepsilon_a)}{1-\alpha}} \tag{22}$$

Given that $b > \frac{1}{\beta\Theta}$, we have that γ_y exceeds γ_h. The presence of a financial system makes the output grow at a rate which is higher than the growth rate of human capital.

5 Calibration

We now present a model calibration based on the US economy's long-run growth rates over the 1950–2019 period aiming to quantitatively assess our main theoretical conclusions. This will help us achieving two separate goals. First, we quantify the effect of endogenous financial literacy on long-term economic growth predicted by our model. Second, we assess the relative magnitude of the two mechanisms outlined in Proposition 3 and determine which one is most likely to dominate in specific circumstances.

Up to now we have considered the financial system to earn a (gross) return $R(y_t, a_t, v_t)$ for every unit of the final good saved and invested, but we have not specified how this return is produced. In order to proceed with the model calibration, we need to explain how financial literacy and income affect $R(y_t, a_t, v_t)$. The mechanism we consider in our calibration works through stock market participation (SMP) and builds on well-established empirical regularities.[11] First, SMP has been showed to be empirically relevant for the US in improving capital market efficiency (see for example Greenwood and Jovanovic, 1990, and Bencivenga and Smith, 1991). Moreover, the main stock market indexes have earned positive real returns on average over sufficiently long investment horizons (e.g. Fama and French, 2002).[12] And finally, there is large empirical evidence that general knowledge (human capital) and specialized knowledge (financial literacy) both contribute positively to stock market participation. Individuals with high financial literacy are more likely to participate in the stock market than those with low financial literacy (van Rooij et al., 2011, 2012, Cole and Shastry, 2009, Yoong, 2011, among others). Financially literate individuals face lower costs for collecting and processing information and thus face a lower economic threshold for stock market participation. Investigations on the effect of human capital (education) on SMP show that college-educated are more likely to own stocks (Haliassos and Bertaut 1995; Campbell, 2006; Lusardi and de Bassa Scheresberg 2013; Cole and Shastry, 2008; Thomas and Spataro, 2018). Putting together all this empirical evidence, we postulate that higher levels of human capital and financial literacy induce higher SMP, so that the gross return generated by the financial system per unit of investment is higher.

Specifically, in our calibration we start from the gross return R as in (21) from Example 1, and consider that R is a function of SMP in a way consistent with the example. In particular, let q_t denote the SMP at period t. We assume that q_t depends on the macroeconomic variables through a Cobb-Douglas relation as follows:

$$q_t(u_t, v_t, h_t, a_t) = \varepsilon_{R,u} u_t^{\epsilon_{q,u}} \varepsilon_{R,v} v_t^{\epsilon_{R,v}} \varepsilon_{R,h} h_t^{\epsilon_{q,h}} \; \varepsilon_{R,a} a_t^{\epsilon_{q,a}}$$

which holds for all t. This allows us to rewrite R_t as:

$$R_t = \frac{{u_t}^{\varepsilon_{R,u}-\epsilon_{q,u}} v_t^{\varepsilon_{R,v}-\epsilon_{q,v}} h^{\varepsilon_{R,h}-\epsilon_{q,h}} a^{\varepsilon_{R,a}-\epsilon_{q,a}} q_t}{\varepsilon_{R,u}\varepsilon_{R,v}\varepsilon_{R,h}\varepsilon_{R,a}}$$

It is straigthforward to show that along the BGP equilibrium $\varepsilon_{R,i} = \frac{dR}{dq}\frac{q}{R}\varepsilon_{q,i} = \epsilon_{q,i}$, for $i = \{u, v, h, a\}$. This means that we can proxy the elasticities of the gross return function with respect to our key variables with the corresponding elasticities computed from stock market participation.

Since we lack information on some key model parameters, in particular on the elasticities of SMP with respect to the flow of new financial literacy (and on the amount of human capital devoted to production) we do not try to directly estimate them from data.[13] Rather, we derive plausible values of the model parameters

[11] There are other possible mechanisms affecting the degree of efficiency of the financial sector, namely credit intermediation and its allocational efficiency, or, broadly speaking, financial development (e.g., Levine, 1997). However, the empirical relationship between financial development and the key variables of our model, i.e. human capital and financial literacy, is still relatively unexplored.

[12] Also the literature on households portfolios shows positive annual returns over various, sufficiently long periods (e.g., Calvet et al., 2007, von Gaudecker, 2014).

[13] For example, values of the elasticity of R with respect to financial literacy, both in terms of stock (a), and flow (v), are difficult to obtain from the empirical literature due to very different existing measures of financial literacy, the relatively short time horizon covered by panel data on financial literacy, and the difficulty to disentangle stocks and flows.

from the existing literature and match the BGP growth rates of output and human capital predicted by our model with the observed ones. This procedure gives us the calibrated values of the unknown elasticities.

In our baseline parametrization we set the share of physical capital in the final good production α to the very stylized value of 0.4 (e.g., Feenstra et. al., 2015), the discount factor β as 0.9524, which implies an intertemporal discount rate of $\rho = \frac{1}{\beta} - 1 = 0.05$ (Samwick, 1998; Gustman and Steinmeier, 2005; Bozio et al., 2017), and the productivity of human capital in human capital accumulation b as 1.057, which is consistent with the average of the estimates for developed countries by Wedel (2022). The intensity of the financial literacy stock in the production of new financial literacy ξ is arbitrarily set as 0.6, and we show in Appendix A that our conclusions are robust to different values of this parameter. The elasticity of SMP with respect to human capital $\epsilon_{q,h} = \varepsilon_{R,h}$ is instead set as 1.089, consistent with the estimates by Hong et al. (2005), who regress stock market participation on several controls, including human capital.[14] The output and human capital growth rates, γ_y and γ_h, are calculated from the PWT 10.0 data as 1.98% and 0.53%, respectively (Feenstra et al., 2015).[15]

We determine the remaining elasticities of SMP $\epsilon_{q,a} = \varepsilon_{R,a}$ and $\epsilon_{q,u} + \epsilon_{q,v} = \varepsilon_{R,u} + \varepsilon_{R,v}$ to match the observed output and human capital growth rates by solving the system of two equations (16) and (22) for these parameters. In particular, for a given value of $\varepsilon_{R,h}$ and γ_h there exists a unique value of $\varepsilon_{R,a}$ satisfying (22). By plugging this result in (16) we obtain the value $\varepsilon_{R,u} + \varepsilon_{R,v}$ that matches the observed growth rate of output. Table 1 reports the parameter values employed in our analysis and the results of our matching procedure.

α	β	b	ξ	$\epsilon_{R,h}$	$\epsilon_{R,a}$	$\epsilon_{R,u} + \epsilon_{R,v}$
0.4	0.9524	1.057	0.6	1.089	2.92	1.24

Table 1: Employed and derived parameter values in our calibration.

The calibrated elasticities $\varepsilon_{R,a}$ and $\varepsilon_{R,u} + \varepsilon_{R,v}$ take the values 2.92 and 1.24, respectively. The high value for the elasticity $\varepsilon_{R,a}$ suggests that, in order to match the real growth rates of human capital and output, financial literacy should have a very strong effect on SMP in our model. This is consistent with van Rooij et al. (2011), who find that a one-standard deviation increase in financial literacy raises SMP by about 9%, which in their model would imply an elasticity of SMP with respect to financial literacy approximately equal to 1.70.

We are not aware instead of any work aiming to estimate the elasticity of SMP with respect to education and financial literacy investments, thus we are unable to compare our calibrated value for $\epsilon_{R,u} + \epsilon_{R,v}$ with previous studies. Nevertheless, our baseline results consistently confirm that SMP and the financial return largely depend on financial literacy, both in terms of stock and flow investment. Specifically, both $\varepsilon_{R,a}$ and $\varepsilon_{R,u} + \varepsilon_{R,v}$ are larger than one. This implies that even small increases in financial literacy (both in terms of stock and flow due to financial education) will result in sizeable increases in the average long-term return of the financial system, which in turn promotes faster growth by making more resources available for production. Such a conclusion is in line with Lusardi et al.'s (2017) who were the first to show that financial literacy matters for macroeconomic outcomes. While Lusardi et al. (2017) assess the importance of financial knowledge for wealth inequality, our result suggests that financial literacy plays an important role for long-run economic growth.

The calibrated elasticities reported in Table 1 allow us to derive another interesting conclusion. Since $\varepsilon_{R,u} + \varepsilon_{R,v} > \varepsilon_{R,h}$, it follows that in our baseline calibration the financial return channel prevails over the human capital one (see Proposition 3). This suggests that the beneficial effect of the financial system on economic growth is mainly driven by the accumulation of financial knowledge and how this translates into

[14] Hong et al. (2005) define stock market participation as in the Health and Retirement Study, which asks respondents whether they own stocks either directly or through mutual funds. They measure human capital as years of education.

[15] Note that these parameter values are consistent with Example 1 as the condition $\gamma_y > \gamma_h$ is verified.

increased SMP, rather than by human capital formation. Even if the acquisition of financial literacy drains resources away from formal education and therefore reduces long run growth, such an effect is more than compensated by the improved ability of the financial system to intertemporally transfer savings. Therefore, not only we can conclude that financial literacy matters for long run economic growth but we can also explain the reason why it matters. The positive relation between financial knowledge and stock market participation along with the relation between SMP and the average long-run stock market returns determines the speed of capital accumulation and output growth.

5.1 Robustness and Sensitivity Analysis

In order to assess the robustness of our results we now consider a wider range of possible values for $\varepsilon_{R,a}$ and for $\varepsilon_{R,u}+\varepsilon_{R,v}$ and we analyze how the growth rates predicted by our model vary with different combinations of these elasticities.

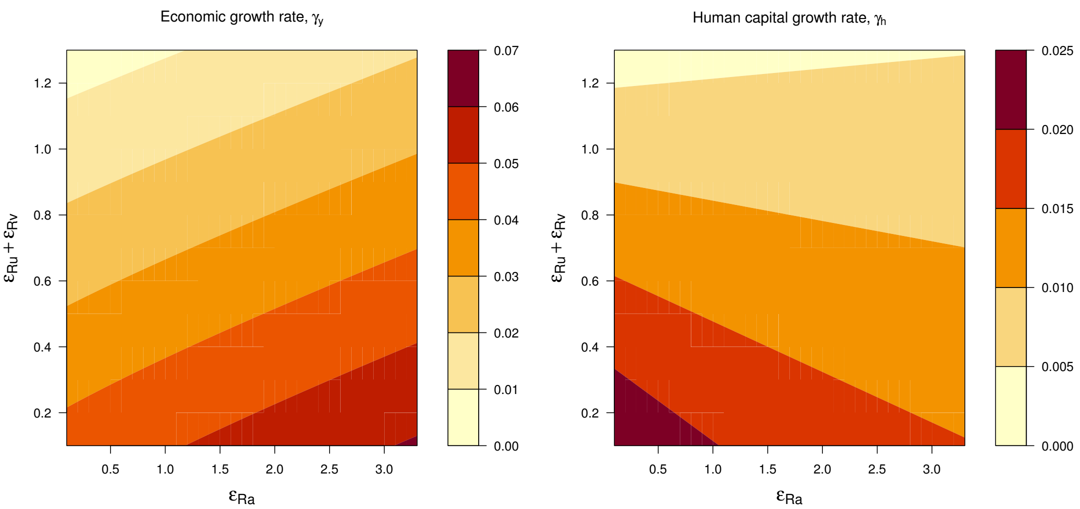

Figure 1: The relation between either the growth rate of per capita output, γ_y, (left) or the growth rate of per capita output, γ_h, (right) and the key unobserved elasticities of our model ($\epsilon_{R,a}$ and $\epsilon_{R,u}+\epsilon_{R,v}$, respectively).

Figure 1 presents the results of our analysis for the output (left panel) and human capital (right panel) growth rates. As it may be reasonable to expect, we can observe that both the growth rates increase with $\varepsilon_{R,a}$ and decrease with $\varepsilon_{R,u}+\varepsilon_{R,v}$. Indeed, $\varepsilon_{R,a}$ positively affects growth for two reasons: it induces higher human capital accumulation as education favors financial literacy acquisition (see equation (2)), and at the same time makes the financial return more sensitive to the existing stock of financial literacy.

The effects of $\varepsilon_{R,u}+\varepsilon_{R,v}$ on the growth rates are instead more complicated as this parameter works like a "double-edge sword". On the one hand, higher $\varepsilon_{R,u}+\varepsilon_{R,v}$ may yield higher return on investment in the financial markets because it favors human capital formation and financial literacy accumulation. On the other hand, financial literacy accumulation reduces human capital formation. Our results suggest that this latter mechanisms prevails on the former so that the output and human capital growth rates decrease with $\varepsilon_{R,u}+\varepsilon_{R,v}$.

We now perform some sensitivity analysis to assess whether and how our results change with some key parameter values. Our model builds on five exogenous parameters: the elasticity of the return function with respect to human capital $\varepsilon_{R,h}$, the physical capital share in the output technology α, the elasticity of financial literacy production with respect to its existing stock ξ, the discount factor β, and the productivity of education in human capital formation b. We show (see Appendix A) that the first three parameters have a very limited impact on our calibration results, and therefore we focus here on the effects of β and b. The results of our sensitivity analysis for the discount factor (left panel) and for the productivity of education in human capital formation (right panel) are reported in Figure 2.

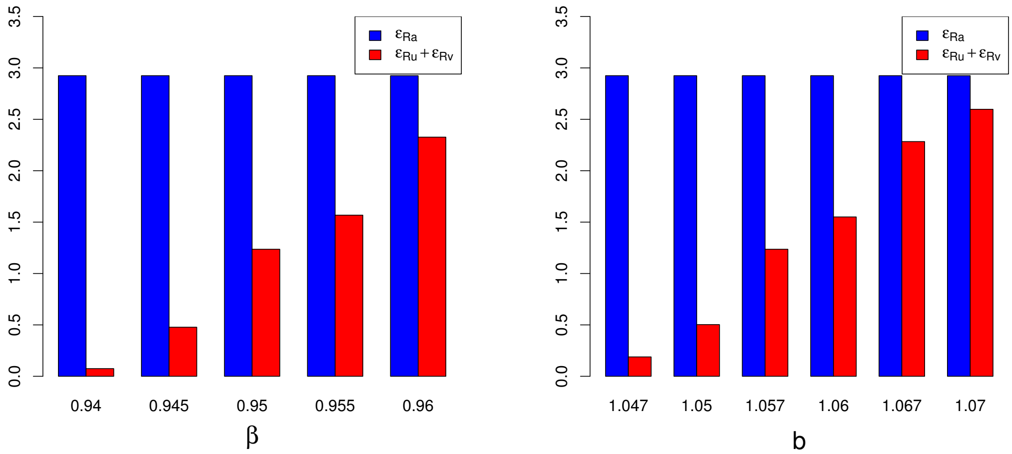

Figure 2: Sensitivity analysis for different values of β (left) and b (right).

Consistent with some studies which use lower and upper bounds for the discount rate $\rho = \frac{1}{\beta} - 1$ of nearly 0.04 and 0.11 (Samwick, 1998; Gustman and Steinmeier, 2005; Bozio et al., 2017), we vary β within the interval (0.94, 0.96) to ensure that the condition $\varepsilon_{R,u} + \varepsilon_{R,v} \geq 0$ in Proposition 1 holds true. The calibrated $\varepsilon_{R,a}$ turns out to be quite insensitive to changes in β, while $\varepsilon_{R,u} + \varepsilon_{R,v}$ increases with it (see the left panel of Figure 2). In order to match the observed growth rates, an increase (a decrease) in impatience, that is a reduction (an increase) in β, requires an increase (a decrease) in the incentive to accumulate human capital rather than financial literacy. This is possible if $\varepsilon_{R,u} + \varepsilon_{R,v}$ decreases (increases) relatively to $\varepsilon_{R,h}$. Given the estimated value of $\varepsilon_{R,h} = 1.089$ (Hong et al., 2015), both the financial return and the human capital channel may prevail according to the value of the discount factor. In particular, the financial return channel (human capital channel) dominates for high (low) values of β when impatience is relatively low (high). Intuitively, this is because a very patient representative agent (high β) makes high investments in human capital, so that the human capital growth rate γ_h is relatively high. In order to match the observed growth rates of both human capital and output, one needs a sufficiently high $\varepsilon_{R,v}$ that gives the representative agent enough incentives to accumulate financial literacy at the expenses of human capital.

We then vary the productivity of education b within the interval (1.047, 1.7) – see e.g. Funke and Strulik (2000). The calibrated $\varepsilon_{R,a}$ turns out to be quite insensitive to changes in b, while $\varepsilon_{R,u} + \varepsilon_{R,v}$ strongly increases with it (see the right panel of Figure 2). An increase (a decrease) in the productivity of education leads to an increase (a decrease) in the return to human capital investment. In order to match the observed growth rates, our model requires an increase (a decrease) in financial education, which is possible only if $\varepsilon_{R,v}$ increases (decreases). As before for the discount factor β, both the financial return and the human capital channel may prevail according to the value of the productivity of education b. The financial return channel (human capital channel) dominates for high (low) values of b, when accumulating human capital is relatively simple (difficult).

6 Conclusions

This paper studies the impact of financial literacy on long-run equilibrium economic growth. We develop an extension of a human-capital-based multi-sector endogenous growth model with the addition of a financial system that transfers savings intertemporally. Our extension consists in postulating that individuals may purposefully invest resources to increase not only their level of general education (human capital), but also their level of financial literacy (defined as knowledge and skills specific to the financial sector). Moreover, we also maintain that the presence of a financial system seeking to improve resource-allocation at the macroeconomic level translates into an increase in the efficiency of physical capital investment. Financial literacy is an intangible asset that requires both financial education and general education (human capital) to be augmented. However, financial literacy does not affect the production of the final consumption good, but (together with other macroeconomic variables) contributes to increasing the efficiency with which, at the aggregate level, a financial system may convert units of current saving into units of future physical capital.

We show that the possibility to improve the efficiency with which savings can be transferred intertemporally dramatically changes the results with respect to an analogous setup (e.g., Uzawa-Lucas) where such a possibility is precluded. By generating a dynamic tradeoff concerning the allocation of existing human capital, the presence of a financial system affects the long-term growth rate of the economy and the characteristics of its BGP equilibrium. On the cost side, the presence of a financial system accruing some return from its activity of resource-allocation increases the opportunity cost of time for human capital accumulation. On the benefit side, it also provides incentives to accumulate financial literacy if this has a positive effect on the return of the financial activity itself.

Based on this tradeoff, we identify two distinct ways through which finance (in the sense specified above, i.e. any activity able to increase the efficiency of the technology that transforms savings in units of new physical capital) can benefit long-term economic growth. When the return on savings earned by the financial system is highly sensitive to financial education, agents invest in financial education and reduce their human capital accumulation. Provided that the improvement in the financial efficiency is sufficiently strong, then the growth rate of the economy exceeds the one in the Uzawa-Lucas model, and along the BGP the relative size of the different economic sectors will change across time (the “financial return channel”). Alternatively, finance contributes to amplifying economic growth when the return from its allocative activity strongly depends on human capital. In this case human capital grows faster than in the Uzawa-Lucas setup, increasing economic growth (the “human capital channel”).

In order to quantify the effect of endogenous financial knowledge on long-term economic growth predicted by our model, we perform a calibration based on the experience of the US economy over the 1950–2019 period. In our calibration we exploit the well-known empirical evidence that both financial literacy and human capital increase stock market participation. Our model is able to replicate the observed growth rates of human capital and output if stock market participation is highly sensitive to the degree of financial literacy and to the investment in financial education.

To the best of our knowledge, ours is the first attempt to analyze the implications of financial literacy on long run economic growth, and we have analyzed this by exploring the mutual relations among human capital, financial education and the efficiency of the financial system. However, financial literacy and financial education may affect growth through other mechanisms such as lowering the uncertainty associated with investments or increasing the availability of resources for research and development activities. In order to better understand the possible effects of financial literacy on long run macroeconomic outcomes it is thus important to extend the analysis along these directions. This is left for future research.

A Sensitivity Analysis

In order to further study the sensitivity of our baseline results, we study the effects of a change in the elasticity $\varepsilon_{R,h}$. This parameter plays a key role because it provides the benchmark value against which $\varepsilon_{R,u} + \varepsilon_{R,v}$ is compared in order to quantitatively assess the two channels described in Proposition 3. We refer again to Hong et al. (2005), but now we move away from their point estimate of years of education on SMP (implying our baseline value $\varepsilon_{R,h} = \varepsilon_{q,h} = 1.089$) by considering the maximum and the minimum values of their estimates. The results are illustrated in Figure 3. We can observe that the calibrated $\varepsilon_{R,a}$ and $\varepsilon_{R,u} + \varepsilon_{R,v}$ vary very little with $\varepsilon_{R,h}$, and therefore the qualitative conclusions from our baseline analysis are robust to reasonable changes in the value of $\varepsilon_{R,h}$.

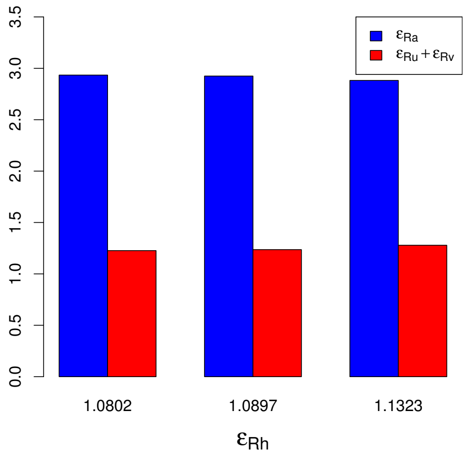

Figure 3: Sensitivity analysis for different values of $\epsilon_{R,h}$ according to the maximum and minimum coefficient value in Hong et.al. (2005): 0.228 and 0.239.

We also analyze the effects of a change in the physical capital share in the output technology α and the elasticity of financial literacy production with respect to its existing stock ξ. The results from this analysis are shown in Figure 4 for α (left panel) and ξ (right panel). Changes in the parameter α affect only slightly the calibrated value of $\varepsilon_{R,u} + \varepsilon_{R,v}$, while $\varepsilon_{R,a}$ is increasing in α. When α rises, this increases the importance of the physical capital growth. For a given growth rate of output, we need a higher $\varepsilon_{R,a}$ to induce a faster accumulation of financial literacy, that in turns produces higher returns on the financial market, hence more physical capital growth. Concerning the parameter ξ, the elasticities $\varepsilon_{R,a}$ and $\varepsilon_{R,u} + \varepsilon_{R,v}$ are quite independent of it. While $\varepsilon_{R,a}$ is not affected by ξ, $\varepsilon_{R,u} + \varepsilon_{R,v}$ slightly decreases with ξ.

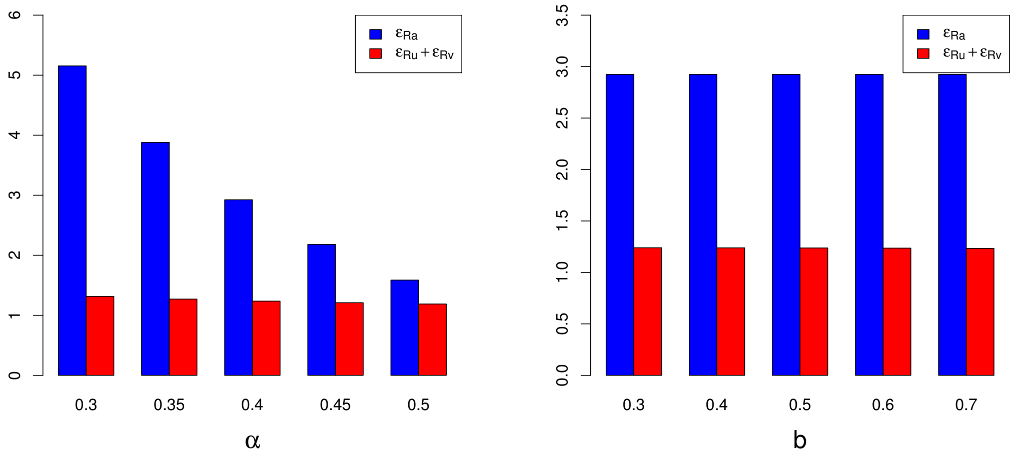

Figure 4: Robustness check to different values of α (left) and ξ (right).

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B Technical Appendix [ONLINE ONLY]

B.1 Proof of Proposition 1

The Bellman equation associated with problem (5) reads as follows:

$$V(k_t, h_t, a_t) = \max_{c_t, u_t, \nu_t} \{\ln c_t + \beta V(k_{t+1}, h_{t+1}, a_{t+1})\}$$

We look for an explicit expression of the value function by applying the "guess and verify" method. We therefore conjecture the following functional form:

$$V(k_t, h_t, a_t) = \theta + \theta_k \ln k_t + \theta_h \ln h_t + \theta_a \ln a_t$$

where θ_i with $i = \{k, h, a\}$ are parameters to be determined. From the above expression, after substituting the dynamic constraints, the Bellman equation reads as:

$$\begin{aligned} \theta + \theta_k \ln k + \theta_h \ln h + \theta_a \ln a \quad = \quad & \max_{c,u,\nu} \left\{ \ln c + \beta\theta + \beta\theta_k \ln \left(R(.)(k^\alpha (uh)^{1-\alpha} - c) \right) + \beta\theta_h \ln \left(b(1-u-\nu)h \right) \right. \\ & \left. + \beta\theta_a \ln \left((\nu h)^{1-\xi} a^\xi \right) \right\} \end{aligned} \tag{23}$$

The FOCs follow:

$$\frac{1}{c} = \frac{\beta\theta_k}{k^\alpha (uh)^{1-\alpha} - c} \tag{24}$$

$$\beta\theta_k \left(\frac{\varepsilon_{R,u}}{u} + \frac{(1-\alpha)k^\alpha (uh)^{-\alpha} h}{k^\alpha (uh)^{1-\alpha} - c} \right) = \frac{\beta}{1-u-\nu}\theta_h \tag{25}$$

$$\frac{1}{\nu}\left(\theta_k \varepsilon_{R,\nu} + \theta_a (1-\xi) \right) = \theta_h \frac{1}{1-u-\nu} \tag{26}$$

where $\varepsilon_{R,u} = \frac{\frac{\partial R}{\partial u} u}{R}$, and $\varepsilon_{R,\nu} = \frac{\frac{\partial R}{\partial \nu} \nu}{R}$. The envelope conditions imply:

$$\frac{\theta_k}{k} = \left(\beta\varepsilon_{R,k} + \frac{\alpha\beta k^\alpha (uh)^{1-\alpha}}{k^\alpha (uh)^{1-\alpha} - c} \right) \frac{\theta_k}{k} \tag{27}$$

$$\frac{\theta_h}{h} = \beta\theta_k \left(\frac{(1-\alpha)k^\alpha (uh)^{-\alpha} u}{k^\alpha (uh)^{1-\alpha} - c} + \varepsilon_{R,h} \frac{1}{h} \right) + \beta\theta_h \frac{1}{h} + \beta\theta_a (1-\xi) \frac{1}{h} \tag{28}$$

$$\theta_a = \frac{\beta\varepsilon_{R,a}}{1-\beta\xi}\theta_k \tag{29}$$

where $\varepsilon_{R,k} = \frac{\frac{\partial R}{\partial k} k}{R}$. From (24) and (27) we obtain:

$$c = \frac{1-\alpha\beta-\beta\varepsilon_{R,k}}{1-\beta\varepsilon_{R,k}} y \tag{30}$$

$$\theta_k = \frac{\alpha}{1-\alpha\beta-\beta\varepsilon_{R,k}}, \tag{31}$$

while from (29) and (31):

$$\theta_a = \frac{\alpha\beta\varepsilon_{R,a}}{(1-\beta\xi)(1-\alpha\beta-\beta\varepsilon_{R,k})} \tag{32}$$

From (28), after plugging (24) and (31), we get:

$$\theta_h = \frac{(1-\alpha)\left(1-\alpha\beta-\beta\varepsilon_{R,k}\right) + \alpha\beta\left(1-\alpha+\varepsilon_{R,h} + \frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)}{(1-\beta)\left(1-\alpha\beta-\beta\varepsilon_{R,k}\right)} \tag{33}$$

The three expressions (31), (32), and (33) determine the parameters in the value function V, while the constant θ follows from solving (23) after substituting all relevant expressions.

From (28) and (25), and by using (26), we have:

$$u+\nu=\overline{u}+\overline{\nu}=1-\beta\frac{\theta_h}{\theta_h-\beta\theta_k\left(\varepsilon_{R,h}-\varepsilon_{R,u}-\varepsilon_{R,\nu}\right)} \tag{34}$$

By combining (26) and (25) with (34), after some algebra it is possible to obtain:

$$\nu \;=\; \overline{\nu}=\frac{\beta\left(\theta_k\varepsilon_{R,\nu}+\left(1-\xi\right)\theta_a\right)}{\theta_h-\beta\theta_k\left(\varepsilon_{R,h}-\varepsilon_{R,u}-\varepsilon_{R,\nu}\right)} \tag{35}$$

$$u \;=\; \overline{u}=\frac{\beta\theta_k\varepsilon_{R,u}+\left(1-\alpha\right)\left(1+\beta\theta_k\right)}{\theta_h-\beta\theta_k\left(\varepsilon_{R,h}-\varepsilon_{R,u}-\varepsilon_{R,\nu}\right)} \tag{36}$$

From (33) and (31) we have:

$$1-\overline{u}-\overline{\nu}=\beta\left(1-\frac{\alpha\beta(1-\beta)\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}-\varepsilon_{R,h}\right)}{1-\alpha-\beta\varepsilon_{R,k}+\alpha\beta\left(\varepsilon_{R,k}+\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)+\alpha\beta(1-\beta)\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}-\varepsilon_{R,h}\right)}\right) \tag{37}$$

while from (35) and (36):

$$\frac{\overline{\nu}}{\overline{u}}=\frac{\varepsilon_{R,\nu}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}}{\varepsilon_{R,u}+\frac{1-\alpha}{\alpha\beta}\left(1-\beta\varepsilon_{R,k}\right)} \tag{38}$$

By combining the previous two expression it is possible to determine:

$$\overline{u}=\frac{1-\beta\left(\frac{1-\alpha-\beta\varepsilon_{R,k}+\alpha\beta\left(\varepsilon_{R,k}+\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)}{1-\alpha-\beta\varepsilon_{R,k}+\alpha\beta\left(\varepsilon_{R,k}+\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)+\alpha\beta(1-\beta)\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}-\varepsilon_{R,h}\right)}\right)}{\frac{\varepsilon_{R,u}+\frac{1-\alpha}{\alpha\beta}\left(1-\beta\varepsilon_{R,k}\right)+\varepsilon_{R,\nu}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}}{\varepsilon_{R,u}+\frac{1-\alpha}{\alpha\beta}\left(1-\beta\varepsilon_{R,k}\right)}} \tag{39}$$

which can be written as (7) by defining

$$\Theta \;=\; \frac{1-\alpha-\beta\varepsilon_{R,k}+\alpha\beta\left(\varepsilon_{R,k}+\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)}{1-\alpha-\beta\varepsilon_{R,k}+\alpha\beta\left(\varepsilon_{R,k}+\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)+\alpha\beta(1-\beta)\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}-\varepsilon_{R,h}\right)} \tag{40}$$

$$\Delta \;=\; \frac{\varepsilon_{R,u}+\frac{1-\alpha}{\alpha\beta}\left(1-\beta\varepsilon_{R,k}\right)+\varepsilon_{R,\nu}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}}{\varepsilon_{R,u}+\frac{1-\alpha}{\alpha\beta}\left(1-\beta\varepsilon_{R,k}\right)} \tag{41}$$

We substitute (39) into (38) to obtain

$$\overline{\nu}=\frac{1-\beta\left(\frac{1-\alpha-\beta\varepsilon_{R,k}+\alpha\beta\left(\varepsilon_{R,k}+\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)}{1-\alpha-\beta\varepsilon_{R,k}+\alpha\beta\left(\varepsilon_{R,k}+\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)+\alpha\beta(1-\beta)\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}-\varepsilon_{R,h}\right)}\right)}{\frac{\varepsilon_{R,u}+\frac{1-\alpha}{\alpha\beta}\left(1-\beta\varepsilon_{R,k}\right)+\varepsilon_{R,\nu}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}}{\varepsilon_{R,\nu}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}}}$$

which can be written as (8) by the use of Θ and Δ defined in (40) and (41) respectively.

We now turn to the conditions on the model parameters and on the elasticities such that our solution has economic relevance, i.e. $c_t \geq 0$, $\overline{u} \in (0,1)$ and $\overline{\nu} \in (0,1)$, and $\overline{u}+\overline{\nu} \in (0,1)$.

Suppose that all elasticities $\varepsilon_{R,i} \geq 0$, $i=\{k,h,a,u,\nu\}$. Then from (6) it is immediate to see that $c_t \geq 0$ if $\varepsilon_{R,k} \leq \frac{1-\alpha\beta}{\beta}$.

Let us study first the conditions under which $\overline{u}>0$.

From (7), we have that $\overline{u}>0$ if both $\beta\Theta<1$ and $\Delta>0$, with Θ and Δ in (40) and (41). A sufficient condition for $\Delta>0$ is $\varepsilon_{R,k} \leq \frac{1}{\beta}$, given that all other elasticites $\varepsilon_{R,u}$, $\varepsilon_{R,\nu}$ and $\varepsilon_{R,a}$ are non-negative, and

$\alpha < 1$, $\xi < 1$.

In order to verify that $\Theta < \frac{1}{\beta}$, first notice that a sufficient condition for Θ's numerator to be positive is $\varepsilon_{R,k} \leq \frac{1-\alpha}{\beta} < \frac{1-\alpha\beta}{\beta} < \frac{1}{\beta}$.[16] We proceed by distinguishing two cases:

(i) $\varepsilon_{R,u} + \varepsilon_{R,\nu} \geq \varepsilon_{R,h}$.

The condition $\varepsilon_{R,k} \leq \frac{1-\alpha}{\beta}$ is sufficient for Θ's denominator to be positive, and therefore

$$\Theta = \frac{1-\alpha-\beta\varepsilon_{R,k}+\alpha\beta\left(\varepsilon_{R,k}+\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)}{1-\alpha-\beta\varepsilon_{R,k}+\alpha\beta\left(\varepsilon_{R,k}+\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)+\alpha\beta(1-\beta)\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}-\varepsilon_{R,h}\right)} \leq 1 < \frac{1}{\beta}$$

and $\overline{u} > 0$.

(ii) $\varepsilon_{R,u} + \varepsilon_{R,\nu} < \varepsilon_{R,h}$.

In order to verify that $\Theta < \frac{1}{\beta}$ we first need to determine the sign of Θ's denominator, which is positive if

$$1-\alpha+\alpha\beta(1-\beta)\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}\right)+\alpha\beta^2\left(\varepsilon_{R,h}+\frac{1-\xi}{1-\beta\xi}\varepsilon_{R,a}\right) > \beta\varepsilon_{R,k}\left(1-\alpha\right)$$

The LHS of the above inequality is larger than $(1-\alpha)$ because all $\varepsilon_{R,u}$, $\varepsilon_{R,\nu}$, $\varepsilon_{R,h}$, and $\varepsilon_{R,a}$ are non-negative, while the RHS is lower than $(1-\alpha)$ because $\varepsilon_{R,k} < \frac{1}{\beta}$. Then Θ's denominator is positive. The condition $\Theta < \frac{1}{\beta}$ is equivalent to

$$\left(1-\alpha\right)\left(1-\beta\right)+\alpha\beta\left(1-\beta\right)\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}\right)+\frac{\alpha\beta^2(1-\xi)}{1-\beta\xi}\varepsilon_{R,a} > -\beta^2\left(1-\alpha\right)\varepsilon_{R,k}$$

which always holds, given that $\varepsilon_{R,u}+\varepsilon_{R,\nu} \geq 0$, $\varepsilon_{R,a} \geq 0$ and $\varepsilon_{R,k} \geq 0$.

Summarizing, $\varepsilon_{R,k} \leq \frac{1-\alpha}{\beta}$ is sufficient to guarantee that both $\beta\Theta < 1$ and $\Delta > 0$ and therefore $\overline{u} > 0$.

Now we analyze the conditions under which $\overline{\nu} > 0$.

From (38) we write $\overline{\nu} = \frac{\varepsilon_{R,\nu}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}}{\varepsilon_{R,u}+\frac{1-\alpha}{\alpha\beta}\left(1-\beta\varepsilon_{R,k}\right)}\overline{u}$. Given that $\varepsilon_{R,\nu} \geq 0$, $\varepsilon_{R,a} \geq 0$, and $\varepsilon_{R,k} \leq \frac{1-\alpha}{\beta}$ (so that $\overline{u} > 0$ by the discussion above), one can see that $\overline{\nu} > 0$ if $\varepsilon_{R,u} + \frac{1-\alpha}{\alpha\beta}\left(1-\beta\varepsilon_{R,k}\right) > 0$, which is verified for $\varepsilon_{R,k} < \frac{1}{\beta}$.

Finally, we study the conditions under which both $\overline{u} < 1$ and $\overline{\nu} < 1$.

Having established that both $\overline{u} > 0$, $\overline{\nu} > 0$, a sufficient condition for $\overline{u} < 1$ and $\overline{\nu} < 1$ is $\overline{u} + \overline{\nu} < 1$. By (37),

$$\begin{aligned}
\overline{u}+\overline{\nu} &= 1-\beta\left(1-\frac{\alpha\beta(1-\beta)\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}-\varepsilon_{R,h}\right)}{1-\alpha-\beta\varepsilon_{R,k}+\alpha\beta\left(\varepsilon_{R,k}+\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)+\alpha\beta(1-\beta)\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}-\varepsilon_{R,h}\right)}\right) \\
&= 1-\beta\Theta
\end{aligned} \tag{42}$$

from (40). The condition $\overline{u}+\overline{\nu} < 1$ reduces to $\Theta > 0$. We have shown above that both Θ's numerator and Θ's denominator are both positive when $\varepsilon_{R,k} \leq \frac{1-\alpha}{\beta}$, irrespectively whether $\varepsilon_{R,u}+\varepsilon_{R,\nu} \geq \varepsilon_{R,h}$ or $\varepsilon_{R,u}+\varepsilon_{R,\nu} < \varepsilon_{R,h}$. This concludes the proof that $\overline{u}+\overline{\nu} < 1$.

We now derive the comparative statics results, starting from $\overline{u}$, which we write as in (7), so that, for every $\varepsilon_{R,i}$, $i = \{h, a, u, \nu\}$:

$$\frac{\partial\overline{u}}{\partial\varepsilon_{R,i}} = \frac{-\beta\frac{\partial\Theta}{\partial\varepsilon_{R,i}}\Delta-\left(1-\beta\Theta\right)\frac{\partial\Delta}{\partial\varepsilon_{R,i}}}{\Delta^2} \tag{43}$$

[16] Therefore, the condition $\varepsilon_{R,k} \leq \frac{1-\alpha}{\beta}$ also guarantees that $c_t \geq 0$ and that $\Delta > 0$.

Let us compute:

$$\frac{\partial\Theta}{\partial\varepsilon_{R,h}}=\frac{\alpha\beta(1-\beta)\left(\alpha\beta\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}\right)+\alpha\beta\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}+1-\alpha-\beta\varepsilon_{R,k}+\alpha\beta\varepsilon_{R,k}\right)}{\left(1-\alpha-\beta\varepsilon_{R,k}+\alpha\beta\left(\varepsilon_{R,k}+\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)+\alpha\beta(1-\beta)\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}-\varepsilon_{R,h}\right)\right)^2}>0$$

given that $\varepsilon_{R,k}\leq\frac{1-\alpha}{\beta}$ and all other elasticities are non-negative, while $\frac{\partial\Delta}{\partial\varepsilon_{R,h}}=0$. From (43), we obtain that $\frac{\partial\overline{u}}{\partial\varepsilon_{R,h}}<0$. Next,

$$\begin{aligned}
\frac{\partial\Theta}{\partial\varepsilon_{R,u}} &< 0\\
\frac{\partial\Delta}{\partial\varepsilon_{R,u}} &= -\frac{\varepsilon_{R,\nu}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}}{\left(\varepsilon_{R,u}+\frac{1-\alpha}{\alpha\beta}\left(1-\beta\varepsilon_{R,k}\right)\right)^2}<0
\end{aligned}$$

and from (43) we can conclude that $\frac{\partial\overline{u}}{\partial\varepsilon_{R,u}}>0$.

Concerning the comparative statics of $\overline{\nu}$, we start by considering again (8)

$$\overline{\nu}=\frac{1-\beta\Theta}{\Delta}\frac{\varepsilon_{R,\nu}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}}{\varepsilon_{R,u}+\frac{1-\alpha}{\alpha\beta}\left(1-\beta\varepsilon_{R,k}\right)}=\frac{1-\beta\Theta}{\frac{\varepsilon_{R,u}+\frac{1-\alpha}{\alpha\beta}\left(1-\beta\varepsilon_{R,k}\right)+\varepsilon_{R,\nu}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}}{\varepsilon_{R,\nu}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}}}=\frac{1-\beta\Theta}{\Delta_1}$$

and, as shown before, $\frac{\partial\Theta}{\partial\varepsilon_{R,h}}>0$, $\frac{\partial\Theta}{\partial\varepsilon_{R,u}}=\frac{\partial\Theta}{\partial\varepsilon_{R,\nu}}<0$. From $\frac{\partial\overline{\nu}}{\partial\varepsilon_{R,i}}=\frac{-\beta\frac{\partial\Theta}{\partial\varepsilon_{R,i}}\Delta_1-(1-\beta\Theta)\frac{\partial\Delta_1}{\partial\varepsilon_{R,i}}}{\Delta_1^2}$ we obtain

$$\frac{\partial\overline{\nu}}{\partial\varepsilon_{R,h}}=\frac{-\beta\frac{\partial\Theta}{\partial\varepsilon_{R,h}}\Delta_1}{\Delta_1^2}<0$$

because Δ_1 is independent of $\varepsilon_{R,h}$, and

$$\frac{\partial\overline{\nu}}{\partial\varepsilon_{R,\nu}}=\frac{-\beta\frac{\partial\Theta}{\partial\varepsilon_{R,\nu}}\Delta_1-(1-\beta\Theta)\frac{\partial\Delta_1}{\partial\varepsilon_{R,\nu}}}{\Delta_1^2}>0$$

because

$$\frac{\partial\Delta_1}{\partial\varepsilon_{R,\nu}}=\frac{\varepsilon_{R,\nu}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}-\left(\varepsilon_{R,u}+\frac{1-\alpha}{\alpha\beta}\left(1-\beta\varepsilon_{R,k}\right)+\varepsilon_{R,\nu}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)}{\left(\varepsilon_{R,\nu}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)^2}=-\frac{\varepsilon_{R,u}+\frac{1-\alpha}{\alpha\beta}\left(1-\beta\varepsilon_{R,k}\right)}{\left(\varepsilon_{R,\nu}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)^2}<0$$

We also derive the comparative statics w.r. to $\varepsilon_{R,a}$. Starting from $\frac{\partial\overline{u}}{\partial\varepsilon_{R,a}}=\frac{-\beta\frac{\partial\Theta}{\partial\varepsilon_{R,a}}\Delta-(1-\beta\Theta)\frac{\partial\Delta}{\partial\varepsilon_{R,a}}}{\Delta^2}$ and

$$\begin{aligned}
\frac{\partial\Theta}{\partial\varepsilon_{R,a}} &= \frac{(\alpha\beta)^2\frac{\beta(1-\xi)(1-\beta)}{1-\beta\xi}\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}-\varepsilon_{R,h}\right)}{\left(1-\alpha-\beta\varepsilon_{R,k}+\alpha\beta\left(\varepsilon_{R,k}+\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)+\alpha\beta(1-\beta)\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}-\varepsilon_{R,h}\right)\right)^2}\\
\frac{\partial\Delta}{\partial\varepsilon_{R,a}} &= \frac{\frac{\beta(1-\xi)}{1-\beta\xi}}{\left(\varepsilon_{R,u}+\frac{1-\alpha}{\alpha\beta}\left(1-\beta\varepsilon_{R,k}\right)\right)^2}>0
\end{aligned}$$

one has

$$\frac{\partial\overline{u}}{\partial\varepsilon_{R,a}}=\begin{cases}<0\text{ if }\frac{\partial\Theta}{\partial\varepsilon_{R,a}}>0\text{, i.e. if }\varepsilon_{R,u}+\varepsilon_{R,\nu}<\varepsilon_{R,h}\\ <0\text{ if }\frac{\partial\Theta}{\partial\varepsilon_{R,a}}=0\text{, i.e. if }\varepsilon_{R,u}+\varepsilon_{R,\nu}=\varepsilon_{R,h}\\ \text{not decidable if }\frac{\partial\Theta}{\partial\varepsilon_{R,a}}<0\text{, i.e. if }\varepsilon_{R,u}+\varepsilon_{R,\nu}>\varepsilon_{R,h}\end{cases}$$

and for $\overline{\nu}=\frac{1-\beta\Theta}{\Delta_1}$:

$$\frac{\partial \Delta_1}{\partial \varepsilon_{R,a}} = \frac{-\frac{\beta(1-\xi)}{1-\beta\xi}\left(\varepsilon_{R,u}+\frac{1-\alpha}{\alpha\beta}\left(1-\beta\varepsilon_{R,k}\right)\right)}{\left(\varepsilon_{R,\nu}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)^2} < 0$$

if $\varepsilon_{R,u} + \frac{1-\alpha}{\alpha\beta}\left(1 - \beta\varepsilon_{R,k}\right) \geq 0$ (which is guaranteed by $\varepsilon_{R,k} > \frac{1}{\beta}$). Therefore, $\frac{\partial \overline{\nu}}{\partial \varepsilon_{R,a}} = \frac{-\beta\frac{\partial \Theta}{\partial \varepsilon_{R,a}}\Delta_1 - (1-\beta\Theta)\frac{\partial \Delta_1}{\partial \varepsilon_{R,a}}}{\Delta^2}$

$$\frac{\partial \overline{\nu}}{\partial \varepsilon_{R,a}} = \begin{cases} > 0 \text{ if } \frac{\partial \Theta}{\partial \varepsilon_{R,a}} < 0, \text{ i.e. if } \varepsilon_{R,u} + \varepsilon_{R,\nu} > \varepsilon_{R,h} \\ > 0 \text{ if } \frac{\partial \Theta}{\partial \varepsilon_{R,a}} = 0, \text{ i.e. if } \varepsilon_{R,u} + \varepsilon_{R,\nu} = \varepsilon_{R,h} \\ \text{not decidable if } \frac{\partial \Theta}{\partial \varepsilon_{R,a}} > 0, \text{ i.e. if } \varepsilon_{R,u} + \varepsilon_{R,\nu} < \varepsilon_{R,h} \end{cases}$$

■

B.2 Proof of Proposition 2

The fact that both the share of human capital allocated to production, u_t and to financial literacy accumulation, ν_t, are constant simply derives from (7)-(8) in Proposition 1, with $\varepsilon_{R,k} = 0$:

$$\begin{aligned} \overline{u} &= \frac{1-\beta\Theta}{\Delta} \\ \overline{\nu} &= \frac{1-\beta\Theta}{\Delta} \frac{\varepsilon_{R,\nu} + \frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}}{\varepsilon_{R,u} + \frac{1-\alpha}{\alpha\beta}} \end{aligned}$$

Notice that $\varepsilon_{R,k} = 0 \leq \frac{1-\alpha}{\beta}$, given that $\alpha < 1$. Therefore, by Proposition 1, we have that $\overline{u} \in (0,1)$, $\overline{\nu} \in (0,1)$ and $\overline{u} + \overline{\nu} < 1$.

Again by Proposition 1 (see (12)) one obtains $\frac{h_{t+1}}{h_t} = 1 + \gamma_h = b(1 - \overline{u} - \overline{\nu})$. Therefore the long-run rate of growth of human capital in our model γ_h is positive only if $b > \frac{1}{1-\overline{u}-\overline{\nu}}$. The expression for $\overline{u} + \overline{\nu}$ can be obtained from (42) with $\varepsilon_{R,k} = 0$:

$$\begin{aligned} \overline{u} + \overline{\nu} &= 1 - \beta\left(1 - \frac{\alpha\beta(1-\beta)\left(\varepsilon_{R,u} + \varepsilon_{R,\nu} - \varepsilon_{R,h}\right)}{1-\alpha+\alpha\beta\left(\varepsilon_{R,h} + \frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right) + \alpha\beta(1-\beta)\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}-\varepsilon_{R,h}\right)}\right) \\ &= 1 - \beta\Theta \end{aligned}$$

hence

$$1 - (\overline{u} + \overline{\nu}) = \beta\Theta \tag{44}$$

and therefore

$$\gamma_h = b\beta\Theta - 1$$

We have $\gamma_h > 0$ if $b > \frac{1}{\beta\Theta'}$.

It is easy to verify that a_t grows at the same rate as human capital, $\gamma_a = \gamma_h$ when $\overline{\nu} > 0$:

$$1 + \gamma_a = \frac{a_{t+1}}{a_t} = \frac{\overline{\nu}^{1-\xi} h_t^{1-\xi} a_t^{\xi}}{\overline{\nu}^{1-\xi} h_{t-1}^{1-\xi} a_{t-1}^{\xi}} = \left(\frac{h_t}{h_{t-1}}\right)^{1-\xi} \left(\frac{a_t}{a_{t-1}}\right)^{\xi} = (1+\gamma_h)^{1-\xi}(1+\gamma_a)^{\xi}$$

which holds only if $\gamma_a = \gamma_h$, when both γ_h and γ_a are strictly positive.

Let γ_k be the rate of growth of physical capital k_t. By using $u_t = \overline{u}$ and the budget constraint for physical capital in (4), one obtains

$$1 + \gamma_k = \frac{k_{t+1}}{k_t} = \frac{R_t \alpha\beta \overline{u}^{1-\alpha} k_t^{\alpha} h_t^{1-\alpha}}{R_{t-1}\alpha\beta\overline{u}^{1-\alpha} k_{t-1}^{\alpha} h_{t-1}^{1-\alpha}}$$

where R_t is the return on savings generated at time t, i.e. $R(h_t, a_t, \overline{u}, \overline{\nu})$, while R_{t-1} is the return on savings generated at $t-1$, i.e. $R(h_{t-1}, a_{t-1}, \overline{u}, \overline{\nu})$. Therefore:

$$\begin{aligned} 1 + \gamma_k &= \frac{R_t}{R_{t-1}} \left(\frac{k_t}{k_{t-1}}\right)^{\alpha} \left(\frac{h_t}{h_{t-1}}\right)^{1-\alpha} \\ 1 + \gamma_k &= \frac{R_t}{R_{t-1}} (1+\gamma_k)^{\alpha} (1+\gamma_h)^{1-\alpha} \end{aligned}$$

and solving for $1 + \gamma_k$:

$$1 + \gamma_k = \left(\frac{R_t}{R_{t-1}}\right)^{\frac{1}{1-\alpha}} (1+\gamma_h) = (1+\gamma_R)^{\frac{1}{1-\alpha}} (1+\gamma_h) \tag{45}$$

where γ_R is the growth rate of the financial system' return.

Given that the return function $R(h,a,\overline{u},\overline{\nu})$ is strictly increasing in all its arguments, $\gamma_R \geq 0$ as long as h_t and a_t are non-decreasing. We have obtained above that $\gamma_a = \gamma_h > 0$ if $b > \frac{1}{\beta\Theta}$, and therefore under this condition $\gamma_R \geq 0$. From (45) one derives that under the same condition $b > \frac{1}{\beta\Theta}$, $\gamma_k > 0$.

As for the growth rate of production γ_y (and consumption, since with $\varepsilon_{R,k} = 0$ the optimal consumption $c_t = (1-\alpha\beta)y_t$):

$$\begin{aligned} 1+\gamma_y &= \frac{y_t}{y_{t-1}} = \frac{\overline{u}^{1-\alpha}k_t^{\alpha}h_t^{1-\alpha}}{\overline{u}^{1-\alpha}k_{t-1}^{\alpha}h_{t-1}^{1-\alpha}} = \left(\frac{k_t}{k_{t-1}}\right)^{\alpha}\left(\frac{h_t}{h_{t-1}}\right)^{1-\alpha} \\ &= (1+\gamma_k)^{\alpha}(1+\gamma_h)^{1-\alpha} \\ &= \left(\frac{R_t}{R_{t-1}}\right)^{\frac{\alpha}{1-\alpha}}(1+\gamma_h) = (1+\gamma_R)^{\frac{\alpha}{1-\alpha}}(1+\gamma_h) \end{aligned} \tag{46}$$

by using (45).

Concerning the comparative statics of γ_h w.r. to the elasticities of R, from (16) we have:

$$\frac{\partial\gamma_h}{\partial\varepsilon_{R,i}} = b\beta\frac{\partial\Theta}{\partial\varepsilon_{R,i}}$$

for all $\varepsilon_{R,i} = \{\varepsilon_{R,h}, \varepsilon_{R,a}, \varepsilon_{R,u}, \varepsilon_{R,\nu}\}$. Moreover,

$$\frac{\partial\Theta}{\partial\varepsilon_{R,h}} = \frac{\alpha\beta^2\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}\right)+\alpha\beta\left(1-\alpha+\alpha\beta\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)}{\left(1-\alpha+\alpha\beta\left(\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)+\alpha\beta(1-\beta)\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}-\varepsilon_{R,h}\right)\right)^2} > 0$$

while it is immediate to see from (9) that $\frac{\partial\Theta}{\partial\varepsilon_{R,u}} = \frac{\partial\Theta}{\partial\varepsilon_{R,\nu}} < 0$.

Concerning the elasticity $\varepsilon_{R,a}$, we have:

$$\frac{\partial\Theta}{\partial\varepsilon_{R,a}} = \frac{\alpha\beta(1-\beta)\frac{\alpha\beta^2(1-\xi)}{1-\beta\xi}}{\left(1-\alpha+\alpha\beta\left(\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)+\alpha\beta(1-\beta)\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}-\varepsilon_{R,h}\right)\right)^2}\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}-\varepsilon_{R,h}\right)$$

whose sign corresponds to the sign of $\varepsilon_{R,u}+\varepsilon_{R,\nu}-\varepsilon_{R,h}$. Therefore:

$$\begin{aligned} \frac{\partial\gamma_h}{\partial\varepsilon_{R,a}} &\geq 0 \text{ if } \varepsilon_{R,u}+\varepsilon_{R,\nu} \geq \varepsilon_{R,h} \\ \frac{\partial\gamma_h}{\partial\varepsilon_{R,a}} &< 0 \text{ if } \varepsilon_{R,u}+\varepsilon_{R,\nu} < \varepsilon_{R,h} \end{aligned}$$

We start from (46) to study the comparative statics of γ_y w.r. to the elasticities of R. For any elasticity $\varepsilon_{R,i}$, $i = \{h, a, u, \nu\}$, we have:

$$\begin{aligned} \frac{\partial\gamma_y}{\partial\varepsilon_{R,i}} &= \frac{\alpha}{1-\alpha}(1+\gamma_R)^{\frac{\alpha}{1-\alpha}-1}\frac{\partial\gamma_R}{\partial\varepsilon_{R,i}}(1+\gamma_h) + (1+\gamma_R)^{\frac{\alpha}{1-\alpha}}\frac{\partial\gamma_h}{\partial\varepsilon_{R,i}} \\ &= (1+\gamma_R)^{\frac{\alpha}{1-\alpha}}\left(\frac{\alpha}{1-\alpha}\frac{1+\gamma_h}{1+\gamma_R}\frac{\partial\gamma_R}{\partial\varepsilon_{R,i}} + \frac{\partial\gamma_h}{\partial\varepsilon_{R,i}}\right) \end{aligned} \tag{47}$$

where $\gamma_R = \frac{R_t}{R_{t-1}-1}$ for any $t \geq 0$ and $\frac{\partial\gamma_h}{\partial\varepsilon_{R,i}} = b\beta\frac{\partial\Theta'}{\partial\varepsilon_{R,i}}$. Without loss of generality, let us write $R_t = f(h_t, a_t, \overline{u}, \overline{\nu})$ where $f(.)$ is a standard neoclassical production function, monotone increasing and strictly concave in all its arguments. Then, $R_t = f(b(1-\overline{u}-\overline{\nu})h_{t-1}, (\overline{\nu}h_{t-1})^{1-\xi}\, a_t^{\xi}, \overline{u}, \overline{\nu})$, while $R_{t-1} = f(h_{t-1}, a_{t-1}, \overline{u}, \overline{\nu})$. Then:

$$\frac{\partial\gamma_R}{\partial\varepsilon_{R,i}} = \frac{\partial}{\partial\varepsilon_{R,i}}\left(\frac{f(b(1-\overline{u}-\overline{\nu})h_{t-1}, (\overline{\nu}h_{t-1})^{1-\xi}\, a_{t-1}^{\xi}, \overline{u}, \overline{\nu})}{f(h_{t-1}, a_{t-1}, \overline{u}, \overline{\nu})}\right)$$

for any $t \geq 1$. By considering h_{t-1} and a_{t-1} given, and the function $f(.)$ constant across time for any $d\varepsilon_{R,i}$ one obtains

$$\begin{aligned}
\frac{\partial \gamma_R}{\partial \varepsilon_{R,i}} &= \frac{\partial}{\partial \varepsilon_{R,i}} \left(f(b(1-\overline{u}-\overline{\nu})h_{t-1}, (\overline{\nu}h_{t-1})^{1-\xi}\, a_{t-1}^{\xi}, \overline{u}, \overline{\nu}) \right) \\
&= \frac{\partial f}{\partial h}\frac{\partial b(1-\overline{u}-\overline{\nu})h_{t-1}}{\partial \varepsilon_{R,i}} + \frac{\partial f}{\partial a}\frac{\partial (\overline{\nu}h_{t-1})^{1-\xi} a_{t-1}^{\xi}}{\partial \varepsilon_{R,i}} + \frac{\partial f}{\partial u}\frac{\partial \overline{u}}{\partial \varepsilon_{R,i}} + \frac{\partial f}{\partial \nu}\frac{\partial \overline{\nu}}{\partial \varepsilon_{R,i}} \\
&= \frac{\partial f}{\partial h} bh \left(-\frac{\partial \overline{u}}{\partial \varepsilon_{R,i}} - \frac{\partial \overline{\nu}}{\partial \varepsilon_{R,i}} \right) + \frac{\partial f}{\partial a} \left((1-\xi)\, \overline{\nu}^{-\xi} h^{1-\xi} a^{\xi} \frac{\partial \overline{\nu}}{\partial \varepsilon_{R,i}} \right) + \frac{\partial f}{\partial u}\frac{\partial \overline{u}}{\partial \varepsilon_{R,i}} + \frac{\partial f}{\partial \nu}\frac{\partial \overline{\nu}}{\partial \varepsilon_{R,i}}
\end{aligned}$$

by dropping the suffix $t-1$ and considering h_{t-1} and a_{t-1} as given. Rearrange the above equality to obtain:

$$\frac{\partial \gamma_R}{\partial \varepsilon_{R,i}} = \frac{\partial \overline{u}}{\partial \varepsilon_{R,i}} \left(-bh\frac{\partial f}{\partial h} + \frac{\partial f}{\partial u} \right) + \frac{\partial \overline{\nu}}{\partial \varepsilon_{R,i}} \left(\frac{\partial f}{\partial a} (1-\xi) \frac{(\overline{\nu}h)^{1-\xi}\, a^{\xi}}{\overline{\nu}} - bh\frac{\partial f}{\partial h} + \frac{\partial f}{\partial \nu} \right)$$

with all the partial derivatives $\frac{\partial f}{\partial h}$, $\frac{\partial f}{\partial a}$, $\frac{\partial f}{\partial u}$, $\frac{\partial f}{\partial \nu}$ being positive and the signs of $\frac{\partial \overline{u}}{\partial \varepsilon_{R,i}}$ and of $\frac{\partial \overline{\nu}}{\partial \varepsilon_{R,i}}$ given in Proposition 1: $\frac{\partial \overline{u}}{\partial \varepsilon_{R,h}} < 0$, $\frac{\partial \overline{\nu}}{\partial \varepsilon_{R,h}} < 0$, $\frac{\partial \overline{u}}{\partial \varepsilon_{R,u}} > 0$ and $\frac{\partial \overline{\nu}}{\partial \varepsilon_{R,v}} > 0$.

From (47) one can see that if both $\frac{\partial \gamma_R}{\partial \varepsilon_{R,i}}$ and $\frac{\partial \gamma_h}{\partial \varepsilon_{R,i}}$ have the same sign, then $\frac{\partial \gamma_y}{\partial \varepsilon_{R,i}}$ has that same sign.[17] We have shown above that $\frac{\partial \gamma_h}{\partial \varepsilon_{R,h}} > 0$ and that $\frac{\partial \gamma_h}{\partial \varepsilon_{R,u}} = \frac{\partial \gamma_h}{\partial \varepsilon_{R,\nu}} < 0$. Putting all together we can conclude that:

- given that $\frac{\partial \gamma_h}{\partial \varepsilon_{R,h}} > 0$, also $\frac{\partial \gamma_y}{\partial \varepsilon_{R,h}} > 0$ if $\frac{\partial f}{\partial u}$, $\frac{\partial f}{\partial a}$ and $\frac{\partial f}{\partial \nu}$ are sufficiently small. In this case $\frac{\partial \gamma_R}{\partial \varepsilon_{R,h}} > 0$ given that $\frac{\partial \overline{u}}{\partial \varepsilon_{R,h}} < 0$ and $\frac{\partial \overline{\nu}}{\partial \varepsilon_{R,h}} < 0$;

- given that $\frac{\partial \gamma_h}{\partial \varepsilon_{R,u}} < 0$, also $\frac{\partial \gamma_y}{\partial \varepsilon_{R,u}} < 0$ if $\frac{\partial f}{\partial u}$, $\frac{\partial f}{\partial a}$ and $\frac{\partial f}{\partial \nu}$ are sufficiently small and $\frac{\partial \overline{\nu}}{\partial \varepsilon_{R,u}} > 0$. In this case $\frac{\partial \gamma_R}{\partial \varepsilon_{R,u}} < 0$ given that $\frac{\partial \overline{u}}{\partial \varepsilon_{R,u}} > 0$;

- given that $\frac{\partial \gamma_h}{\partial \varepsilon_{R,\nu}} < 0$, also $\frac{\partial \gamma_y}{\partial \varepsilon_{R,\nu}} < 0$ if $\frac{\partial f}{\partial u}$, $\frac{\partial f}{\partial a}$ and $\frac{\partial f}{\partial \nu}$ are sufficiently small and $\frac{\partial \overline{u}}{\partial \varepsilon_{R,\nu}} > 0$. In this case $\frac{\partial \gamma_R}{\partial \varepsilon_{R,\nu}} < 0$ given that $\frac{\partial \overline{v}}{\partial \varepsilon_{R,\nu}} > 0$.

■

[17] Moreover, the sign of $\frac{\partial \gamma_y}{\partial \varepsilon_{R,i}}$ equals that of $\frac{\partial \gamma_h}{\partial \varepsilon_{R,i}}$ if α is sufficiently close to zero.

B.3 Proof of Proposition 3

We start by recalling the BGP in a standard Uzawa-Lucas model without financial system. The long-run rate of growth of human capital $\frac{h_{t+1}}{h_t} = 1 + \gamma_h^{UL} = 1 + \gamma^{UL} = b(1 - \overline{u}) = b\beta$, given that $1 - \overline{u} = \beta$, with $\gamma^{UL} > 0$ if $b > \frac{1}{\beta}$.

Let us fix the same b across the two models. From (12) and (44), we have that in our framework $1 + \gamma_h = b\beta\Theta$, and $\gamma_h > 0$ if $b > \frac{1}{\beta\Theta'}$. It is straightforward to see that $\gamma_h > \gamma^{UL}$ ($\gamma_h \leq \gamma^{UL}$) if $\Theta > 1$ ($\Theta \leq 1$). For simplicity, let us define $\Gamma = 1 - \Theta$, that is:

$$\Gamma = 1 - \frac{1-\alpha+\alpha\beta\left(\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)}{1-\alpha+\alpha\beta\left(\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)+\alpha\beta(1-\beta)\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}-\varepsilon_{R,h}\right)} = \frac{\alpha\beta(1-\beta)\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}-\varepsilon_{R,h}\right)}{1-\alpha+\alpha\beta\left(\varepsilon_{R,h}+\frac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right)+\alpha\beta(1-\beta)\left(\varepsilon_{R,u}+\varepsilon_{R,\nu}-\varepsilon_{R,h}\right)}$$

We have that $\gamma_h > \gamma^{UL}$ ($\gamma_h \leq \gamma^{UL}$) if $\Gamma < 0$ ($\Gamma \geq 0$).

Let us study the sign of Γ. We start from its denominator. Given that all elasiticites are non-negative and $\alpha < 1$, Γ's denominator is always positive if $\varepsilon_{R,u} + \varepsilon_{R,\nu} - \varepsilon_{R,h} \geq 0$. If $\varepsilon_{R,u} + \varepsilon_{R,\nu} - \varepsilon_{R,h} < 0$ we have that

$$\begin{aligned}
1 - \alpha + \alpha\beta\left(\varepsilon_{R,h} + \tfrac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a}\right) + \alpha\beta(1-\beta)\left(\varepsilon_{R,u} + \varepsilon_{R,\nu} - \varepsilon_{R,h}\right) &> 0 \\
1 - \alpha + \alpha\beta\tfrac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a} + \alpha\beta^2\varepsilon_{R,h} &> -\alpha\beta(1-\beta)\left(\varepsilon_{R,u} + \varepsilon_{R,\nu}\right)
\end{aligned}$$

A sufficient condition for the above inequality to be satisfied is

$$\tfrac{\beta(1-\xi)}{1-\beta\xi}\varepsilon_{R,a} + \beta\varepsilon_{R,h} > 0$$

because $1 - \alpha > 0$ and both $\varepsilon_{R,u}$ and $\varepsilon_{R,\nu}$ are non-negative. This always holds because also $\varepsilon_{R,a}$ and $\varepsilon_{R,h}$ are non-negative. Therefore, we can conclude that the denominator of Γ is positive, and that $sgn(\Gamma) = sgn(\varepsilon_{R,u} + \varepsilon_{R,\nu} - \varepsilon_{R,h})$. Summarizing:

- if $\varepsilon_{R,u} + \varepsilon_{R,\nu} \geq \varepsilon_{R,h}$ then $\Gamma \geq 0$ and $\gamma_h \leq \gamma^{UL}$;
- if $\varepsilon_{R,u} + \varepsilon_{R,\nu} < \varepsilon_{R,h}$ then $\Gamma < 0$ and $\gamma_h > \gamma^{UL}$.

Let us move to the comparison of γ_y with γ^{UL}. Given (18), it is straightforward to verify that $\gamma_y > \gamma^{UL}$ as long as $\gamma_R \geq 0$ and $\gamma_h > \gamma^{UL}$, that is when $\varepsilon_{R,u} + \varepsilon_{R,\nu} < \varepsilon_{R,h}$.

If $\varepsilon_{R,u} + \varepsilon_{R,\nu} \geq \varepsilon_{R,h}$ then $\gamma_h \leq \gamma^{UL}$ and we have that $\gamma_y \geq \gamma^{UL}$ only if

$$1 + \gamma_y = (1 + \gamma_R)^{\frac{\alpha}{1-\alpha}}(1 + \gamma_h) \geq 1 + \gamma^{UL}$$

We substitute for $1 + \gamma_h = b\beta\Theta$ obtained above and for $\gamma^{UL} = b\beta - 1$:

$$\begin{aligned}
(1 + \gamma_R)^{\frac{\alpha}{1-\alpha}} b\beta\Theta &\geq b\beta \\
1 + \gamma_R &\geq \left(\frac{1}{\Theta}\right)^{\frac{1-\alpha}{\alpha}}
\end{aligned}$$

that proves the proposition.[18] ■

[18]Recall that $\Theta' < 1$ if $\varepsilon_{R,u} + \varepsilon_{R,\nu} > \varepsilon_{R,h}$. In this case, $\gamma_h < \gamma^{UL}$ and in order to obtain that $\gamma_y > \gamma^{UL}$ we need that γ_R is "sufficiently high" ($1/\Theta' > 1$).