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# Applying real options to port infrastructure expansion: the case of a Brazilian port

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## Abstract

This case study applies Monte Carlo-based real options to analyse the expansion of a port. A stochastic model is defined for the Santarém port, in Brazil, and the optimal moment for expanding the terminal is determined. The application resorts to a common spreadsheet and a simulation add-in, allowing the quantification of both the value of expanding the terminal and the flexibility to determine when to expand. The results allow us to conclude that deterministic cash flow models, based on the expected value, may lead to important biases in projects with capacity constraints. We also conclude that the expansion option may have a high value, which is strongly determined by the initial conditions, and that the expansion thresholds (the values of demand that trigger the expansion) change significantly along the project.

**Keywords:** Ports; Simulation; Real Options

JEL classification: C63; G31; R42

## 1. Introduction

Real options analysis has been extensively applied to infrastructure investment, sometimes resorting to Monte Carlo simulation. For example, De Neufville et al. (2006) propose a spreadsheet model to evaluate real options related to the capacity of a car park, Godinho and Dias (2012) use a simulation model to analyse the optimal timing of a road infrastructure, Zhao et al. (2004) propose a simulation model to evaluate real options in highway development, operation, and rehabilitation. In many infrastructure projects, capacity constraints occur, making the expansion option particularly valuable. In this case, some cash flows depend on both the demand and the production constraint. Authors that applied real options to projects with capacity constraints suggest that real options analysis provides a more accurate measure of the value of the project, compared to the traditional Net Present Value (NPV) analysis (e.g., Vaughn, 2015).

Several authors have addressed the investment in port infrastructure, sometimes using simulation models. For example, Gómez-Fuster and Jiménez (2020) use simulation to perform a risk analysis of port terminal investment and Lin et al. (2014) develop a simulation model to determine the optimal investment plan for the Humen Port. Some authors analyse port expansion. For example, Chen and Liu (2016) address the optimal expansion decisions by two competing ports and Luo et al. (2012) identify the necessary conditions for a port to increase its profit through capacity expansion.

The use of real options in port infrastructure appraisal has also been the subject of some research. Balliauw et al. (2019) analyse the option to delay investment in a capacity level under demand uncertainty and Cournot-type competition. Balliauw et al. (2020) study the optimal size and timing of the investment in new port capacity. Meersman (2005) focuses on optimizing port investments in an uncertain environment. These works are based on stochastic models that assume that either the demand or the project value follow a geometric Brownian motion. Monte

Carlo simulation provides a tool to build more general models, without restrictive assumptions about the behaviour of key variables. Zheng and Negenborn (2017) used least squares Monte Carlo to address terminal investment decisions when a port faces competition from its rivals in an uncertain market. Lagoudis et al. (2014) develop an Excel simulation model for port expansion.

This paper presents a case study of real options applied to the expansion of a terminal of the Santarém port, in Brazil, using a Monte Carlo model to determine the optimal timing for the expansion of a terminal. Similar to most real options models, and in accordance to the recommendations of several finance textbooks (e.g., Brealey et al., 2016), the analysis is based on the NPV. To build the case study, we define a stochastic model that identifies and corrects the potential bias in deterministic cash flow models. This bias due to Jensen's inequality (see, e.g., Geltner and De Neufville, 2018, p. 39), and capacity constraints, which lead to asymmetric probability distributions of the cash flows, may worsen it. The structure of the model allows it to be used without resorting to unreasonable assumptions about the behaviour of key variables, which are often used in order to obtain close-form solutions for models – for example, key variables are often constrained to follow either a geometric Brownian motion (as in Balliauw et al., 2019, 2020, and Meersman, 2005), a mean reverting Ornstein-Uhlenbeck process or a Poisson jump process. This case study uses a realistic and adaptable model without such constraints, in which the processes followed by relevant variables are defined using past data.

To determine the optimal expansion timing, we use a methodology that is related to De Neufville et al. (2006), Geltner, De Neufville (2018) and Godinho and Dias (2012). De Neufville et al. (2006) consider a simple rule, which they do not optimize, to define when the expansion option is exercised, while Geltner and De Neufville (2018) consider that the real option exercise thresholds are the same for different years. By contrast, we allow the expansion thresholds to change along the years, following the logic that it is more difficult to justify an

expansion investment close to the end of the project, and optimize the values of those thresholds using an expansion timing methodology. This way, this work is closer to Godinho and Dias (2012). However, the different nature of the available data and the difference between the considered options lead to distinct modelling challenges (e.g., considering constraints to the served demand when the underlying demand is stochastic, and considering an expansion investment which is made when the project is already underway) and a different type of analysis. The integrated model and methodology constitute an approach that can be implemented using a common spreadsheet and a simulation add-in.

The paper is structured as follows. After this introduction, the concession model of the Santarém port and its main characteristics are presented. Then, the model for port capacity expansion is explained. The fourth section describes the methodology employed to determine the optimal rules for making the expansion decision. In the fifth section, the results are presented. Finally, the main conclusions are highlighted in the last section.

## **2. The case study: Santarém port**

The concession model is often used in Brazil to attract the private initiative, which pays a rent for exploring these infrastructures. The official method to determine the rent value is defined by the Brazilian Waterway Transportation Agency, ANTAQ. One of the ports managed by ANTAQ is the Santarém port, located in the Pará state. This port is partly used to transport soybean. Soybean grains arrive by waterway and road, coming mainly from the state of Mato Grosso. They are stored and later exported through the Amazonas river and the Atlantic Ocean.

In 2010, an analysis was performed to the “16” and “1 south” areas of the Santarém port, to appraise the construction of a new terminal, mainly for soybean transportation, and a viability study was performed by DTA Engenharia. When the study for the new concession was performed, the Santarém port already had some equipment in place, which was used

mainly for exports. At the time, the port had other lease agreements ongoing. Of these agreements, it is relevant to consider the lease of another terminal to Cargill Agrícola S.A., which was also used for soybean transportation. The new terminal was expected to be built in 2011 and operations were expected to start in 2012. Although several years have already passed, this is still an interesting case study for the evaluation of the expansion option, allowing us to derive some useful insights.

The viability study considered three types of constraints for the capacity of the terminal: load capacity (barges and trucks); berth movements capacity (ships); and storage capacity. The maximum load capacity was estimated to be 3,744,750 tonnes/year for barges and 1,419,120 tonnes/year for trucks, leading to a total capacity of 5,163,870 tonnes/year. The berth annual expedition capacity was estimated to be 6,842,516 tonnes. For storage, the static capacity considered in the study was 95,000 tonnes, including four silos with a capacity of 10,000 tonnes each and a warehouse with 55,000 tonnes. The dynamic capacity is conditioned by the number of “turns” performed by the terminal. A cycle of 10 days was considered to determine the operational capacity of these terminals (ANTAQ, 2009), corresponding to 36 annual “turns” and leading to a dynamic capacity of 3,420,000 tonnes/year (corresponding to 95,000 tonnes x 36 “turns”).

So, according to the viability study, the bottleneck for the terminal capacity was storage. However, the same study mentioned the possibility of investing in a new warehouse, with an additional static storage capacity of 55,000 tonnes. With this expansion, the dynamic storage capacity of the terminal would become 5,400,000 tonnes/year (corresponding to 150,000 tonnes x 36 “turns”). In this case, the bottleneck would be the load capacity, so the terminal capacity would be 5,163,870 tonnes/year.

The viability study mentioned this expansion possibility, but it did not consider it in the appraisal. In the beginning of the terminal operation, the capacity of 3,420,000 tonnes would

be enough to satisfy the expected demand, but after some years it might become an important constraint, seriously limiting the demand that might be served by the terminal.

This case study aims to appraise the new terminal project using a real options approach, considering its possible expansion and the uncertainty underlying the future demand for soybean transportation. To account for this uncertainty, a stochastic model for demand is estimated, based on its historical evolution. This model is the base for a Monte Carlo model of the project, which will be used to evaluate the NPV of the terminal. It is assumed that an expansion decision does not have to be made at the outset, but it may be made according to the way the demand evolves. The demand model and the Monte Carlo model of the project will only use information that was already available when the viability study was done.

The evaluation takes into consideration the interests of both ANTAQ and the company that will operate the concession, without considering how the cash flows will be distributed. The NPV calculation will consider the same time horizon that was used in the viability study and it will be based on information used in that study. It will be considered that the investment in the new terminal entails both building costs (R\$54,849,760.35) and equipment costs (R\$52,836,500.00), corresponding to a total amount of R\$107,686,260.35. For the expansion, investment estimates were obtained considering the new construction and the equipment that will be necessary. Following the accounting rules prevailing in Brazil, straight line depreciation is used: 10 years for equipment (10% per year) and either 25 years (4% per year) or the life of the investment (whichever is shorter) for civil works. For estimating revenues, an amount of R\$15/tonne was used, considering the prices prevailing in Brazil. This is the same price that is used in the viability study and, similarly to that study, it is assumed that prices do not change with demand (implicitly, it is assumed that prices will be determined by the government). Following the same study, costs were separated into fixed and variable costs, allowing a correct adjustment of the total costs to the demand served by the terminal over the years. Two types of

taxes are applied: taxes on revenues, with a total rate of 14.25%, and taxes on profits, with a rate of 25%. For the NPV calculation, ANTAQ established a discount rate of 8,30%. Since an alternative discount rate of 10% was under consideration, the project will also be appraised using that rate.

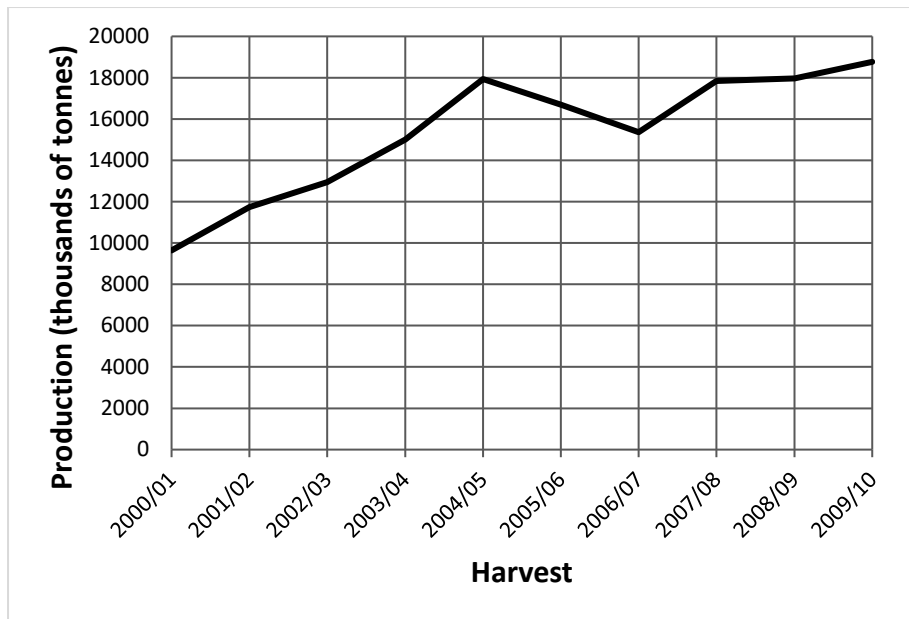
### **3. The model**

The first stage of a Monte Carlo-based evaluation of a project is building a simulation model, in which the cash flows and the NPV are a function of the relevant stochastic factors. In this project, the most relevant stochastic factor concerns the demand for the port terminal. So, a demand model was built and, using this model, a model for the cash flows (and, consequently, the NPV) was defined.

#### ***3.1 The demand model***

Most of the soybean transported from the Santarém port comes from the Mato Grosso state. It is therefore important to analyse behaviour of the production of soybean in this state. Fig. 1 shows the Mato Grosso soybean production, from the 2000/2001 harvest to the 2009/2010 harvest (data available at the time the viability study was made). It is possible to conclude that the soybean production in the Mato Grosso state showed, in this period, an average annual increase of 8.21%, with a standard deviation of 11.18%. These values will be used to help build the demand model.



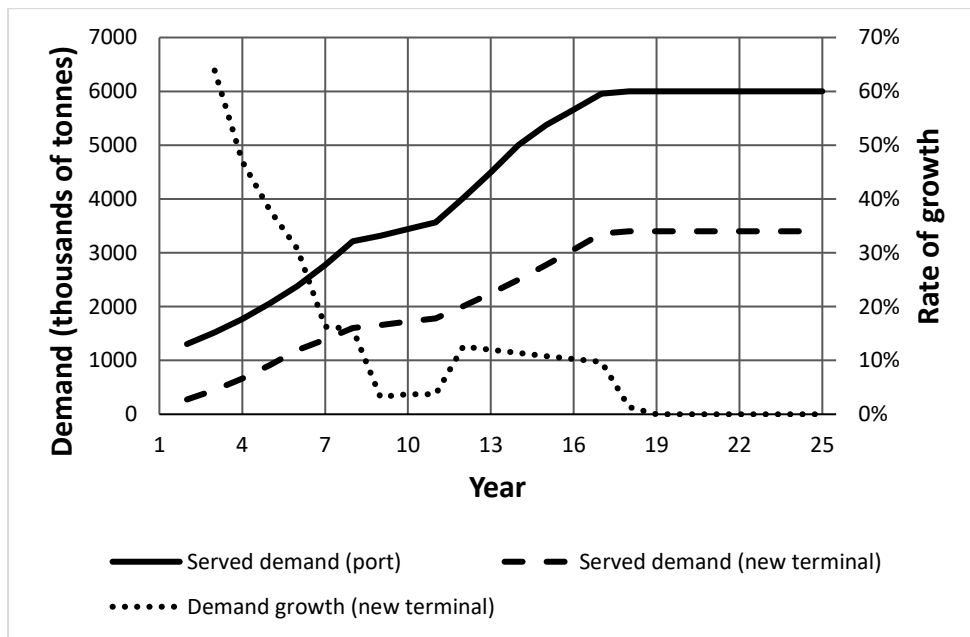


**Fig. 1:** Soybean production in the Mato Grosso state. Source: Conab (2018).

The original viability study for the Santarém port includes deterministic forecasts of the demand for soybean transportation over the years of operation included in the concession. Forecasts are presented both for the new terminal and for the terminal already operated by Cargill. In the case of the new terminal, three demand scenarios (pessimistic, intermediate and optimistic) are defined in the viability study. The demand model is based in the forecasts of the intermediate scenario of the original viability study. These forecasts were used to estimate the expected yearly rate of increase in demand but, since a stochastic model for demand is necessary, the stochastic component still had to be incorporated.

The model considers the total demand of the port and the capacity of the terminals. It is important to distinguish between potential demand and served demand. Potential demand is the quantity of soybean that would be transported if there were no capacity constraints and served demand is the capacity that is effectively transported, considering these constraints. The values of potential and served demand are identical if the potential demand is below the capacity and, otherwise, the served demand is equal to the capacity.

The viability study identifies the expected served demand for each terminal, but the potential demand, which is not provided, will also be necessary to build a stochastic model of demand. The forecasts of the served demand for the port and for the new terminal are depicted in Fig. 2. In this Figure, it is possible to see that the served demand for the new terminal starts at a very low level and, in the initial years, the rate of growth is very high. In the 7<sup>th</sup> year and the following ones, it is assumed that half the port demand goes to the new terminal, so the rate of growth in the demand for the new terminal becomes equal to the rate of growth of the demand for the port. From the 18<sup>th</sup> year onwards, the new terminal is expected to be operating at full capacity, so the rate of growth in served demand is 0%.



**Fig. 2:** Forecasts of the served demand for the Santarém port and for the new terminal, and rate of growth of the demand served by the new terminal. Source: Viability study.

Identifying the existing terminal (already operated by Cargill) by  $i = 0$  and the new terminal by  $i = 1$ , and denoting the potential demand for terminal  $i$  in year  $t$  by  $d_{i,t}^P$ , the served demand by  $d_{i,t}^S$  and the capacity of the terminal by  $c_{i,t}$ , it can be defined that

$$d_{i,t}^S = \min(d_{i,t}^P, c_{i,t}) \quad (1)$$

The total demand for the port is defined as the demand for the two terminals. Denoting the potential demand in year  $t$  by  $D_t^P$ :

$$D_t^P = d_{0,t}^P + d_{1,t}^P \quad (2)$$

The viability study provides an estimate of the expected demand served by each terminal in each year (which is denoted by  $d_{i,t}^{S,e}$ ), assuming that the new terminal is not expanded. From these values, the expected rate of increase in the total demand served by the port in year  $t$  (denoted by  $g_t^{S,e}$ ) can be calculated as

$$g_t^{S,e} = \frac{d_{0,t}^{S,e} + d_{1,t}^{S,e}}{d_{0,t-1}^{S,e} + d_{1,t-1}^{S,e}} - 1 \quad (3)$$

According to the original viability study, the capacity of the Cargill terminal is  $c_{0,t} = 2,600,000$  tonnes/year. The base capacity of the new terminal (without expansion) is  $c_{1,t} = 3,400,000$  tonnes/year and, if/when expansion is undertaken, the capacity becomes  $c_{1,t} = 5,163,870$  tonnes/year. In a deterministic model, it is possible to know when capacity becomes a binding constraint but, in a stochastic model, it is not known at the outset when the capacity limit will be reached. So, the model should be based on the potential demand and capacity constraints should be considered after the potential demand is modelled. The intermediate scenario of the viability study estimates that the existing terminal ( $i = 0$ ) will be working at full capacity from the 15<sup>th</sup> year of the project onwards and the new terminal ( $i = 1$ ) will be operating at full capacity from the 18<sup>th</sup> year onwards. Therefore, from the 15<sup>th</sup> year onwards, the expected potential demand will be higher than the expected served demand and it is no longer possible to use the values of served demand presented in the viability study as expected potential demand. This issue is handled in the following way:

- Until the year 14, when no terminal is expected to be operating at full capacity, the expected growth in the potential demand of the port is assumed to be identical to the expected growth in served demand (defined in (3)).
- From years 15 to 17, the expected rate of increase in the potential demand of the port will be assumed to be equal to the expected rate of growth in the served demand of the new terminal. The viability study assumes that, after the 6<sup>th</sup> year of the project, half of the demand will go to each terminal (apart from capacity constraints). This means that, from year 7 to year 14, the expected growth in the total port demand should be equal to the expected growth in the demand of the new terminal, both for the served and for the potential demand. It seems reasonable to assume that this will still be the case until the new terminal is working at full capacity.
- From year 18 until the end of the project, the expected rate of growth in the potential demand of the port will be assumed to be equal to the historical average rate of increase in the Mato Grosso soybean production, which is 8.21%. The soybean arriving to the port comes mainly from the Mato Grosso state and, moreover, this value is broadly in line with the intermediate scenario of the viability study. In this scenario, the average annual rate of increase in the served demand of the new terminal, in the last ten years for which capacity constraints are not binding (years 8-17) is 9.32%. Since this is the only terminal that is operating without capacity constraints in these years, these values, which are close to the assumed rate of growth of 8.21%, constitute the best reference to be compared with this rate of growth.

Therefore, the expected rate of increase in the total potential demand of the port (denoted by  $g_t^{P,e}$ ) is modelled as

$$g_t^{P,e} = \begin{cases} g_t^{S,e}, & \text{for } t \leq 14 \\ \frac{d_{1,t}^{S,e}}{d_{1,t-1}^{S,e}} - 1, & \text{for } 15 \leq t \leq 17 \\ 8.21\%, & \text{for } t \geq 18 \end{cases} \quad (4)$$

Since the soybean comes mainly from the Mato Grosso state, it is assumed that variations in demand can be approximated by the variability in the production of soybean in this state. So, a stochastic perturbation for the rate of growth of potential demand (denoted by  $\varepsilon_t$ ) is defined as a random variable with mean of zero and a standard deviation equal to the standard deviation in the rate of increase of the soybean production in Mato Grosso, according to historical data for the 2000-2010 period, that is, 11.18%. The series of values of the annual rate of increase in soybean production in 2000-2010 was tested for normality using the Doornik-Hansen, Shapiro-Wilk, Lilliefors and Jarque-Bera tests. None of these tests rejected the null hypothesis of normality at a 10% significance level, so  $\varepsilon_t$  was defined as a normally distributed random variable.

With the expected rate of increase in potential demand for the port,  $g_t^{P,e}$ , and the stochastic term,  $\varepsilon_t$ , the potential demand can be simulated. The simulated rate of increase in potential demand, denoted by  $g_t^{P,sim}$ , is defined as

$$g_t^{P,sim} = g_t^{P,e} + \varepsilon_t, \text{ with } \varepsilon_t \sim Normal(0\%, 11.18\%) \quad (5)$$

Potential demand for the port ( $D_t^{P,sim}$ ) is simulated using the following process:

$$D_t^{P,sim} = D_{t-1}^{P,sim} \cdot (1 + g_t^{P,sim}) \quad (6)$$

It is necessary to split the demand between the two terminals. The viability study assumes that, in years 2 to 6, the largest share of the demand is served by the existing terminal ( $i = 0$ ) and, afterwards, demand is split evenly between the two terminals. Using the values of served demand of the viability study, the share of demand that goes to terminal  $i$  in year  $t$  (denoted by  $f_{i,t}$ ) can be defined as

$$f_{i,t} = \begin{cases} \frac{d_{i,t}^{S,e}}{d_{0,t}^{S,e} + d_{1,t}^{S,e}}, & \text{if } t \leq 6 \\ 50\%, & \text{if } t > 6 \end{cases} \quad (7)$$

For the cash flow model, the simulated demand served by the new terminal,  $d_{1,t}^{S,Sim}$ , is necessary. Having  $D_t^{P,Sim}$  and  $f_{1,t}$ , it can be defined as

$$d_{1,t}^{S,Sim} = \min(f_{1,t} \cdot D_t^{P,Sim}, c_{1,t}) \quad (8)$$

### 3.2 The cash flow and NPV model

As usual with real options models, we use the NPV as the base for project appraisal. The demand model is initially used to simulate demand, and this simulated demand is used to calculate costs and revenues, which are then used to calculate the project cash flows and the NPV. This process is repeated a large number of times (20,000 in this study), allowing us to calculate the expected project NPV. We followed the viability study in the calculation of the cash flows and, similar to the viability study, we used real values for both the cash flows and the discount rate (instead of nominal values – as explained in Brealey et al., 2016, pp. 137-138, both approaches provide consistent results).

Initially, the annual revenues were calculated multiplying the served demand by the unit revenue. Taxes on revenues, costs and depreciations were deducted from the revenues, leading to the calculation of the annual operating profit. Costs were divided into fixed and variable unit costs, allowing a correct adjustment of the total costs to the different values of the served demand. Representing the served demand for the new terminal in year  $t$  by  $d_{1,t}^S$ , the tax rate on revenues by  $tr$  and using the subscript  $t$  to refer to values from year  $t$ , we have:

$$\begin{aligned} \text{Operating profit}_t &= (\text{Unit revenues}_t \cdot (1 - tr) - \text{Unit costs}_t) \cdot d_{1,t}^S - \\ &\quad \text{Fixed Costs}_t - \text{Depreciations}_t \end{aligned} \quad (9)$$

The annual net profit is obtained by removing the profit taxes from the operating profit. Representing the tax rate on profits by  $tp$ , we have:

$$Net\ profit_t = Operating\ profit_t \cdot (1 - tp) \quad (10)$$

The year- $t$  cash flow is then calculated as:

$$C_t = Net\ profit_t + Depreciations_t \quad (11)$$

In the previous expressions, the served demand is determined according to the simulation model presented in Section 3.1 and the other parameters are defined according to the viability study. For each iteration, an NPV is calculated as the sum of the discounted cash flows. The expected project NPV is then defined as the average NPV over all the iterations.

The process described before is straightforward for the case in which no expansion is possible. For the case in which the expansion decision is made dynamically (because it depends of the way that demand evolves in each simulation path), it is necessary to determine, for each simulation path, whether the terminal is expanded and the year in which expansion takes place. In the year in which the expansion investment is made, it is necessary to subtract the corresponding costs from the cash flow and, in the following years, it is necessary to consider the increased depreciations due to this investment. Finally, in the simulation paths in which the expansion investment is made after the 15<sup>th</sup> year of the project, the necessary equipment will still have a strictly positive book value at the end of the project. In those cases, it is assumed that the equipment is sold by a salvage value identical to the remaining book value.

#### **4. Methodology**

From the simulation model, it is necessary to determine the optimal rules for making the expansion decision. The objective is to define rules that allow the maximization of the expected NPV. It will be assumed that, when the decision to expand is made, the corresponding investment is made in the following year and the additional capacity is available two years later.

Denoting the discount rate by  $r$ , the year- $t$  cash flow generated by the terminal by  $C_t$ , and the year- $t$  present value of the terminal by  $V_t^n$  in case it was not expanded before and is not expanded in this year, and by  $V_t^x$  in case the expansion investment is made at year  $t$ :

$$V_t^n = C_t + \frac{1}{1+r} \max[E_t(V_{t+1}^n); E_t(V_{t+1}^x)], \quad (12)$$

where  $E_t(\cdot)$  represents the expected value at year  $t$ .

Expression (12) also indicates how the optimal expansion decision should be made, in case the expansion did not occur before. Since the decision is made in the year before making the investment, the expansion investment should be made at  $t+1$  if  $E_t(V_{t+1}^x) \geq E_t(V_{t+1}^n)$ ; otherwise it is not optimal to expand at  $t+1$ .

The difficulty in applying (12) with a simulation model is related with the estimation of the expected values  $E_t(V_{t+1}^x)$  and  $E_t(V_{t+1}^n)$ . To handle this issue, expansion thresholds will be used to decide whether to expand the terminal. It should be noticed that, within the considered model, the expected values  $E_t(V_{t+1}^x)$  and  $E_t(V_{t+1}^n)$  are defined by the potential demand of the port,  $D_t^P$ . This dependence can be made explicit by writing these expected values as  $E_t(V_{t+1}^x|D_t^P)$  and  $E_t(V_{t+1}^n|D_t^P)$ . An expansion threshold,  $\tau_t$ , is the minimum value of  $D_t^P$  for which it is worthwhile to expand the terminal at year  $t+1$ , that is, the minimum value of  $D_t^P$  for which  $E_t(V_{t+1}^x|D_t^P) \geq E_t(V_{t+1}^n|D_t^P)$ . So, an expansion decision should be made at time  $t$  if and only if  $D_t^P \geq \tau_t$ . Notice that thresholds  $\tau_t$  such that  $E_t(V_{t+1}^x|D_t^P) \geq E_t(V_{t+1}^n|D_t^P)$  if  $D_t^P \geq \tau_t$  and  $E_t(V_{t+1}^x|D_t^P) < E_t(V_{t+1}^n|D_t^P)$  otherwise do exist for the defined simulation model. In fact, the difference  $E_t(V_{t+1}^x|D_t^P) - E_t(V_{t+1}^n|D_t^P)$  is non-decreasing with  $D_t^P$ , so there is a minimum value of  $D_t^P$  for which this difference is non-negative, which is defined as the threshold  $\tau_t$ .

The thresholds  $\tau_t$  can be determined considering that the rules defined by resorting to them allow the maximization of  $V_t^n$ . Consider, for a given realization of the values of the potential demand (that is, for the demand values in a given simulation path), the year- $t+1$  value of the non-expanded terminal, calculated in year  $t$ , defined as a function of a threshold  $\tau$ :



$$V_t(\tau) = \begin{cases} V_{t+1}^x, & \text{if } D_t^p \geq \tau \\ V_{t+1}^n, & \text{if } D_t^p < \tau \end{cases} \quad (13)$$

The year- $t$  threshold,  $\tau_t$ , can be defined as the value of  $\tau$  that maximizes the expected value of  $V_t(\tau)$  over the complete distribution of values of  $D_t^p$ :

$$\tau_t = \underset{\tau}{\operatorname{argmax}} E_t(V_t(\tau)) \quad (14)$$

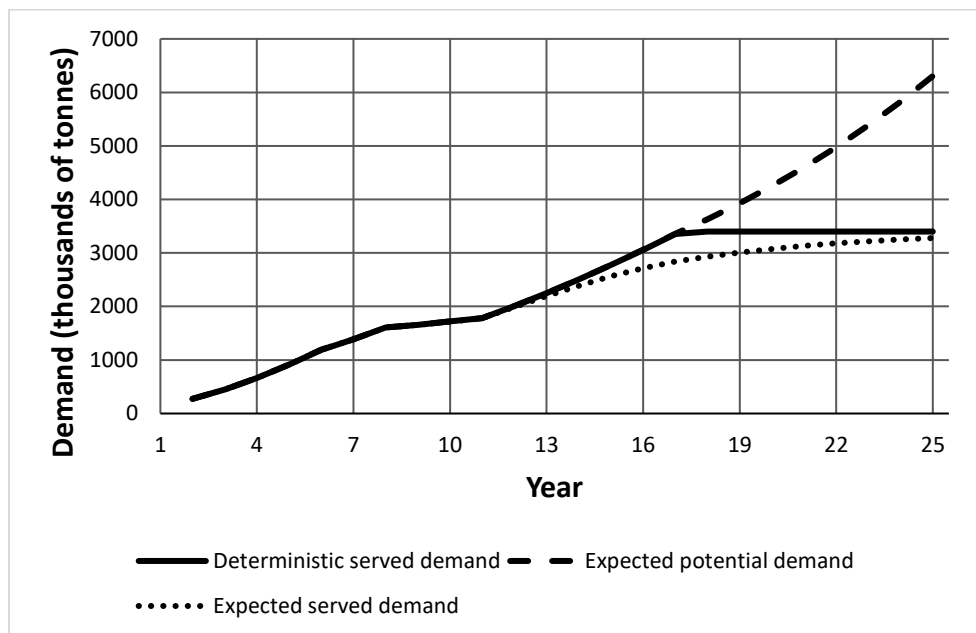
This approach can be implemented in a simulation model, by using a backwards dynamic programming approach. First it is necessary to define the last possible year  $T$  for which it may make sense to decide to expand the terminal. In this case,  $T = 23$ , since if a decision was made in the 24<sup>th</sup> year, the expansion would take place in 25<sup>th</sup> year, the last year of the concession, and the terminal would end up never being used with an expanded capacity.  $\tau_T$  is determined as the value defined by (14), that is, the value that maximizes the average value of  $V_T(\tau)$ , calculated over all simulation paths.  $\tau_{T-1}$  can then be calculated as the value that maximizes the average, over all simulation paths, of  $V_{T-1}(\tau)$ , assuming that, if it is decided not to expand in year  $T - 1$ , a new decision will be made in year  $T$ , considering the threshold  $\tau_T$ . The process then continues until the first year is reached.

As aforementioned, the values  $\tau_t$  are the ones that maximize the average of  $V_t(\tau)$  over all simulation paths. Such values can be determined using a numerical method. In this application, they were maximized within Microsoft Excel using the Solver with the Evolutionary method.

## 5. Results

The simulation models were implemented in Microsoft Excel, with the aid of the simulation add-in Argo (available at <http://boozallen.github.io/argo/>). All simulations comprised 20,000 iterations, because a previous analysis showed that this number of iterations ensured that different simulations led to very similar results.

For reference purposes, the terminal NPV based on the deterministic estimates, and without considering the possibility of expansion, was initially calculated. This calculation led to an NPV of  $R\$25,056 \cdot 10^3$ . The expected NPV was then calculated based on the simulation model, also without considering the possibility of expansion. The NPV thus obtained was  $R\$18,880 \cdot 10^3$ . The large difference between these values may seem somewhat surprising, since the same project is being appraised, with the same assumptions, the only difference being that the first evaluation uses expected values of demand and the second uses simulated values and then calculates the expected value of the obtained NPVs. The problem is that, in the presence of capacity constraints, Jensen's inequality (see, e.g., Geltner and De Neufville, 2018, p. 39) can lead to very large biases. Hence, in these circumstances, cash flows based on expected values of demand do not allow a reliable appraisal of the project. The simulated values of the cash flows provide more accurate estimates of the NPV. We will now show this, based on the demand forecasts of the project being considered.



**Fig. 3:** Deterministic forecasts for the demand served by the new terminal, and expected values of the potential demand and served demand (calculated from the simulation model).

Source: Viability study (deterministic forecast) and authors (expected values).

In Fig. 3, the lines representing the deterministic forecasts of served demand (solid line) and the expected potential demand (dashed line) coincide until the terminal starts operating at full capacity (the difference is never larger than 0.11%, so it is too small to be seen in the Figure). This was to be expected since, as defined by (1), the potential demand should be equal to the served demand until the capacity constraints become binding, and it provides evidence that the stochastic model does not introduce biases in the simulated demand – the expected potential demand is in line with its deterministic estimates, when these are available. However, from the year 12 onwards, the expected served demand calculated with the simulation model (dotted line) is clearly below its deterministic counterpart (solid line), and this is the reason for the difference in the NPVs.

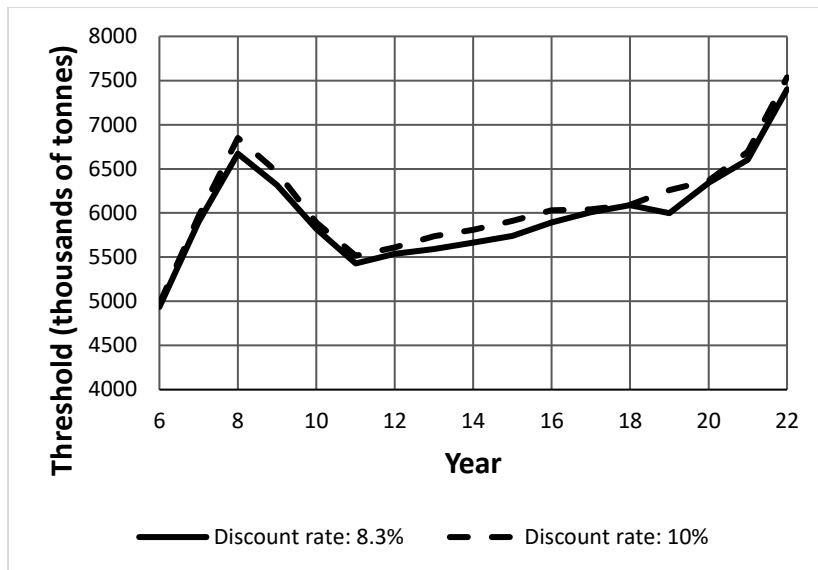
The reason for the difference between the estimates of served demand has to do with the asymmetric nature of the demand uncertainty, in the presence of capacity constraints. Due to uncertainty, potential demand will sometimes be higher than the expected value and other times it will be lower. When it is lower, the reduction in demand will be fully reflected in the cash flows but, when it is higher, it may only be partially reflected (or not at all), due to the capacity constraints. This asymmetry leads to an expected served demand that is lower than its deterministic counterpart, and the difference will be more significant when the potential demand is expected to be close to the capacity limit. Therefore, the deterministic model causes an upward bias in demand estimation, which will be reflected in cash flow and NPV estimation. This bias is not specific to this model. The use of deterministic approximations in the presence of capacity constraints will lead to upward biases in other project models, whenever the company cannot react to changes in demand by changing prices.

After this first analysis, the model was used to evaluate the project including the expansion option. An initial evaluation led to a value of  $R\$26,999 \cdot 10^3$ . However, it was taken

into account that using the same simulated data to determine the thresholds and to apply them to calculate the NPV may lead to an overvaluation of the project – the thresholds thus obtained are over-fitted to the data, leading to a higher project value. So, a new simulation was carried out, using the thresholds estimated with the initial simulation. This new simulation led to an NPV of  $R\$26,989 \cdot 10^3$ . This new value is close to the initial one, probably due to the large number of iterations used in both simulations (20,000), which leaves little space for over-fitting in the threshold estimation of the initial simulation. The expansion option can now be appraised: this option is worth  $R\$26,989 \cdot 10^3 - R\$18,880 \cdot 10^3 = R\$8,109 \cdot 10^3$ , so it is possible to conclude that it significantly contributes to increase the project value.

The project was then appraised considering an alternative discount rate of 10%. Without the expansion option, a deterministic model leads to an NPV of  $R\$3,245 \cdot 10^3$ . However, as explained before, this value will be upwardly biased due to the capacity constraints. A simulation model leads to a negative NPV without the expansion option:  $-R\$1,444 \cdot 10^3$ . If the expansion option is considered, the NPV increases to  $R\$4,208 \cdot 10^3$  (as before, this value is obtained using independent simulations to estimate the thresholds and to determine the NPV). So, if the discount rate increases to 10%, the expansion option becomes even more important. The project is only worthwhile due to this option.

It is also interesting to analyse the behaviour of the expansion thresholds with both discount rates. These thresholds are depicted in Fig. 4. There is no simulation path for which it is worthwhile to expand before year 6 or after year 22, so the thresholds are only shown for the years 6-22.



**Fig. 4:** Expansion thresholds.

With a higher discount rate, higher increases in the cash flows are necessary to make the expansion investment worthwhile. So, with a higher discount rate, the expansion thresholds will be higher, as shown in Fig. 4: the thresholds are close for the two discount rates, but they are slightly higher for the 10% rate. Also, if the expansion investment is made later in the project, a shorter time frame is available for the increased cash flows to make it worthwhile, so only higher cash flows may justify it. Therefore, the thresholds are expected to increase over time. This is the general trend that can be seen in years 6-8 and 11-22 but, in years 8-11, there is a clear decrease in the value of the thresholds. The issue here is that the expected growth in potential demand decreases from values above 15%, up to year 8, to values below 4% in years 9-11, and then it increases to values above 10%. In the simulation model, the low growth in the expected demand in years 9-11 leads to a significant probability of occurrence of a decrease in the demand. Since demand may decrease in years 9-11, a rapid increase in the thresholds occurs before year 9 and, as a period of larger expected growth in demand approaches, the thresholds decrease (due to the previous rapid increase). From the 12<sup>th</sup> year onwards, the general trend of increasing values of the thresholds takes place.

In order to assess the value of the flexibility in choosing when to expand the terminal, the project was appraised assuming that the decision of expanding it at year 17 is made at the outset, without the possibility of being changed. Expansion at the 17<sup>th</sup> year was chosen since this makes the additional capacity available at the year in which it is necessary, according to the expected scenario. The evaluation was made based on the simulation model. With the initial discount rate of 8.3%, an NPV of  $R\$24,942 \cdot 10^3$  was obtained and, with a discount rate of 10%, the NPV was  $R\$2,572 \cdot 10^3$ . These values are higher than the ones obtained assuming that no expansion will be undertaken, but they are clearly lower than the NPVs obtained under the assumption that expansion can be made according to the way the project is developing. So, it may be concluded that the possibility of expansion is valuable, and it becomes even more valuable if there is flexibility in deciding when to undertake it.

## **6. Conclusions**

This paper examines the investment in a port infrastructure using a simulation-based real options model, to determine the optimal timing for the expansion of a port terminal. It is shown that there is a significant difference in the expected NPV of the project when using the simulation model, instead of using the deterministic estimates of demand ( $R\$18,880 \cdot 10^3$  versus  $R\$25,056 \cdot 10^3$ ). Due to asymmetric nature of demand uncertainty, a model based on deterministic estimates overvalues the expected NPV in the presence of capacity constraints, so the best NPV estimate is the one produced with the stochastic model.

When the expansion option is considered, the NPV increases from  $R\$18,880 \cdot 10^3$  to  $R\$26,989 \cdot 10^3$ . So, the expansion option is worth  $R\$8,109 \cdot 10^3$ . For the higher discount rate of 10%, the value of the option becomes even more important, since the NPV would be negative without this option.

This case study addresses the application of real options to a port infrastructure, which has been considered in just a few real options articles, in spite of the importance of port infrastructure for international logistics systems. The applied model allows the identification and correction of potential bias in deterministic cash flow models, when applied to projects with capacity constraints. The model and the methodology can be implemented using a common spreadsheet and a simulation add-in.

There are important practical implications from this case study. The differences in the estimated NPVs with and without the expansion option show the value of flexibility, when concessions include this possibility. When the concession is based on an auction, the value of the option to expand should be considered both by the entity that defines the terms of the auction and by the bidders. Concessions will become more appealing to private companies if they explicitly consider the expansion options. On the one hand, governments can receive a higher rent and attract more bidders, if they design lease agreements that incorporate real options. On the other hand, private companies can make better bids, if they are able to accurately estimate the value of the options that are embedded in lease contracts.

The model presented in this paper can lead to additional studies regarding port infrastructure investments in the presence of uncertainty. Additional issues can be examined in extended models. For example, future research can consider the dynamic volatility of the demand. Port competition and its impact on port infrastructure investments can also be addressed. Although the demand uncertainty is clearly the most important, there are other uncertainties that can be added to the model.

## Notation

*The following symbols are used in this paper:*

$C_t$  = Cash flow generated by the new terminal at year  $t$ ;

$c_{i,t}$  = Capacity of terminal  $i$  in year  $t$ ;

$D_t^P$  = Potential demand of the port in year  $t$ ;

$D_t^{P,sim}$  = Simulated potential demand of the port in year  $t$ ;

$d_{i,t}^P$  = Potential demand for terminal  $i$  in year  $t$ ;

$d_{i,t}^S$  = Demand served by terminal  $i$  in year  $t$ ;

$d_{i,t}^{S,e}$  = Expected demand to be served by terminal  $i$  in year  $t$ ;

$d_{i,t}^{S,sim}$  = Simulated demand served by terminal  $i$  in year  $t$ ;

$E_t(\cdot)$  = Expected value calculated with information available at year  $t$ ;

$f_{i,t}$  = Share of the demand going to terminal  $i$  in year  $t$ ;

$g_t^{P,e}$  = Expected rate of increase in the potential demand of the port in year  $t$ ;

$g_t^{P,sim}$  = Rate of increase in the potential demand of the port simulated for year  $t$ ;

$g_t^{S,e}$  = Expected rate of increase the demand served by the port in year  $t$ ;

$i$  = Terminal;

$r$  = Discount rate;

$T$  = Last possible year for which it may make sense to decide to expand;

$t$  = Year;

$tp$  = Tax rate on profits;

$tr$  = Tax rate on revenue;

$V_t(\tau)$  = Expected present value of the terminal at year  $t$  in case it is not expanded in year  $t$  or before, as a function of the expansion threshold;



$V_t^n$  = Year- $t$  present value of the new terminal in case it is not expanded in year  $t$  or before;

$V_t^x$  = Year- $t$  present value of the new terminal in case it is expanded in year  $t$ ;

$\varepsilon_t$  = Stochastic perturbation for the rate of growth of potential demand in year  $t$ ;

$\tau$  = Threshold;

$\tau_t$  = Threshold defined for year  $t$ .

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